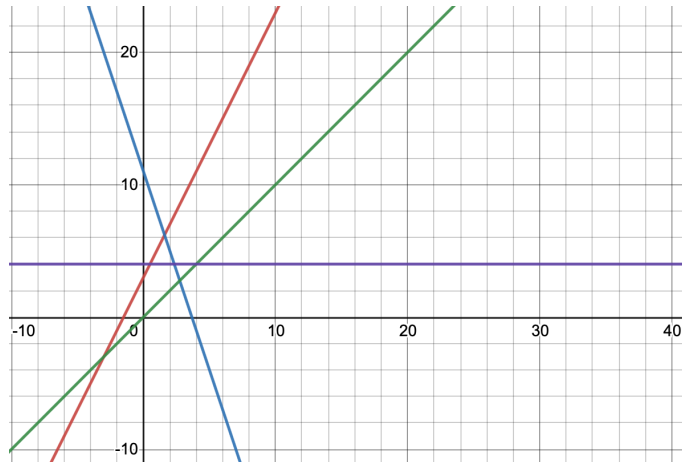# Tables, Equations and Graphs
WORKBOOK ANSWERS

Section One - Part One

1. Understanding Linear Equations

a. A linear equation is a relationship where something increases by a constant amount. When graphed, it looks like a straight line. It has the general form $y = mx + c$, where m is the gradient, and c is a constant (also known as the y-intercept).

b. The gradient of a line can be found by choosing two points and calculating the rise divided by the run for those two points. Because the change is constant, it doesn't matter which two points we choose.

c. The intercepts of a linear equation are where the line passes through the x and y-axis, called the x and y-intercepts. These are the points where $y = 0$ and $x = 0$ respectively. Not all linear equations have these intercepts. One example is when the gradient of the equation is 0 and so the line is horizontal, like $y = 5$ or $y = -31$.

d. Linear equations are good for modelling things that have a constant growth or a constant decrease. Things that they tend to model are things like motion (with a constant acceleration or velocity) or the increase/decrease of something that is changing by the same amount each time (e.g. cost of a good per unit). This is because their equation has a gradient that is constant and the variable is not to the power of anything.

e.

    i. $y = 2x + 3$

        $y = -3x + 11$

        $y = x$

        $y = 4$

    ii.   Linear equations create a straight line on a graph that can either be positive, negative, or a flat line. A positive line is produced from a positive m value (a positive x coefficient) as shown by the red line. A negative line is produced by a negative m value (a negative x coefficient) as shown by the blue line. A flat line is produced when there is no gradient, as shown by the purple line.

         Key features of a linear graph include the gradient (including its direction), the y-intercept, or the constant. Linear graphs will also contain both a y and x variable which become the coordinates of the line at any given point of the relationship.

f.

    i. $y = 3x + 1$

    ii. $y = 4x - 5$

    iii. $y = -3x + 4$

iv.   Red: y = - 7

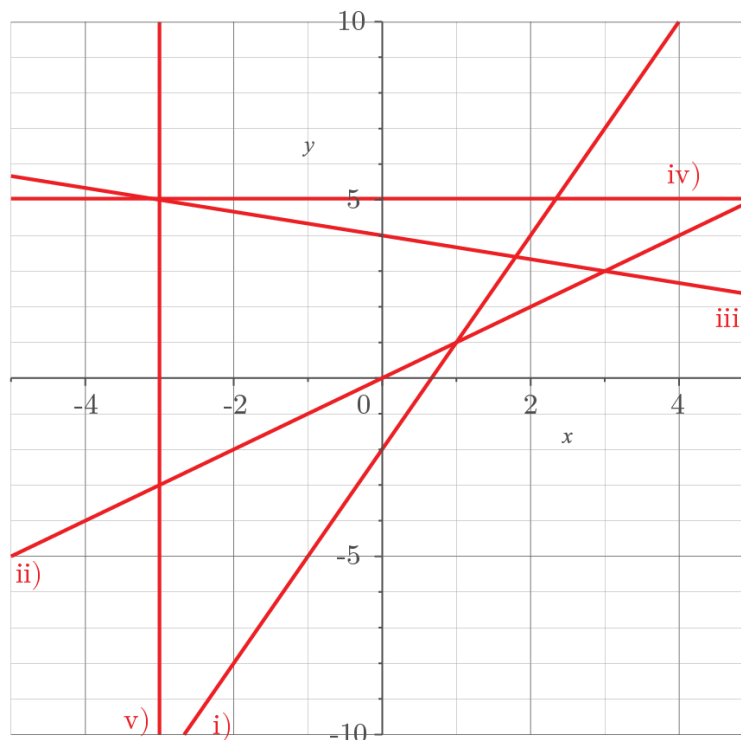 Blue: y = 5

v.   Red: y = $\frac{7}{3}$x

  Blue: y = - 4x + 5

vi.   Red: x = 3

  Blue: y = - 2.5x + 2

Change Note: In previous copies of the workbook, the following questions g. and h. are labelled h. and i. instead.
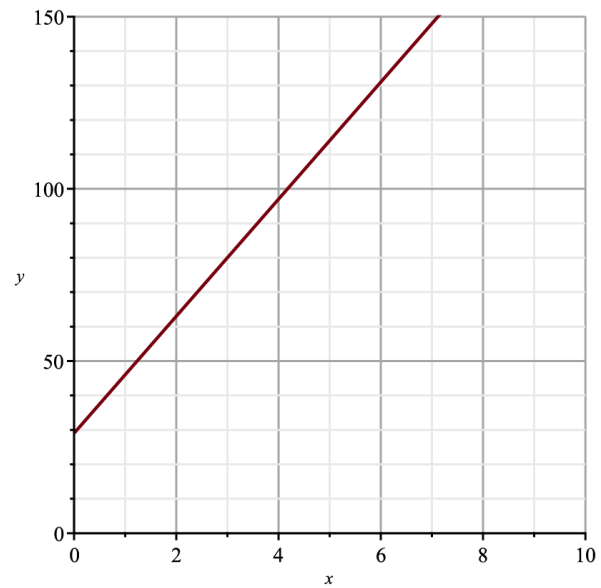

g. To draw a straight line, we choose some values of x to substitute into our equation. We then calculate what y is at each of those x values. We plot each set of coordinates on the graph, and connect them with a straight line. The only exception to this method is v), which does not include the variable y since all values of x are when x=-3, so the line is vertical.

h.

    i. $y = 17h + 29$

h	y	difference
0	29	
1	46	17
2	63	17
3	80	17
4	97	17
5	114	17

    ii.   The gradient of the equation is 17.

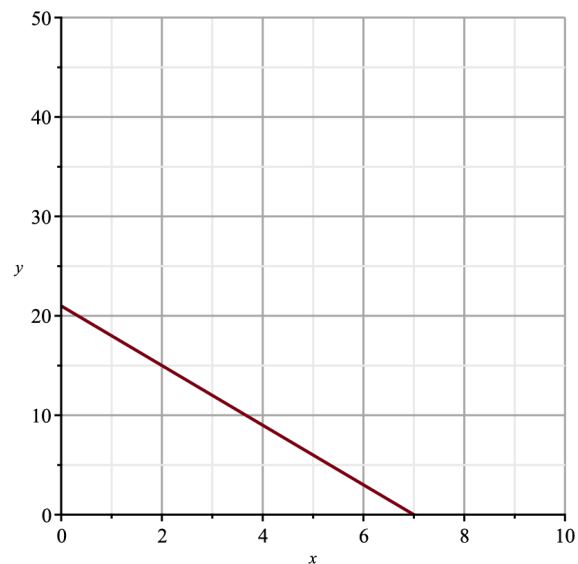To work out the value of y when h is 10, we substitute h=10 into our equation. This gives us:

$y = 17(10)+29$

$y = 170 + 29$

$y = 199$

    iii. $y = -3x + 21$

h	y	difference
3	12	
4	9	-3
5	6	-3
6	3	-3
7	0	-3
8	-3	-3

    iv.   Gradient is -3

$$y = -3x + 21$$
$$y = -3(-1) + 21$$
$$y = 3 + 21$$
$$y = 24$$

2. Understanding Quadratic Equations

a. A quadratic equation includes a variable that is squared. When graphed, they create a parabola. Quadratic equations have the general form:
$y = ax^2 + bx + c$

b. The first form is the factorised form which is $y = k(x-a)(x-b)$, where a and b are the x-intercepts or roots, and k is the scale.
The second form is the vertex form which is $y = k(x-h)^2 + c$, where h is the x coordinate of the vertex (the turning point of a parabola), c is the y-coordinate of the vertex, and k is the scale.

The vertex form would be used when the roots aren't obvious and the factorised form would be used when the coordinates of the vertex aren't obvious. If neither are easy to see, we can use a table instead!

c.

    i. $y = (x+2)(x+1)$

    ii. $y = (x+2)(x-1)$

    iii. $y = -(x+2)^2 + 2$
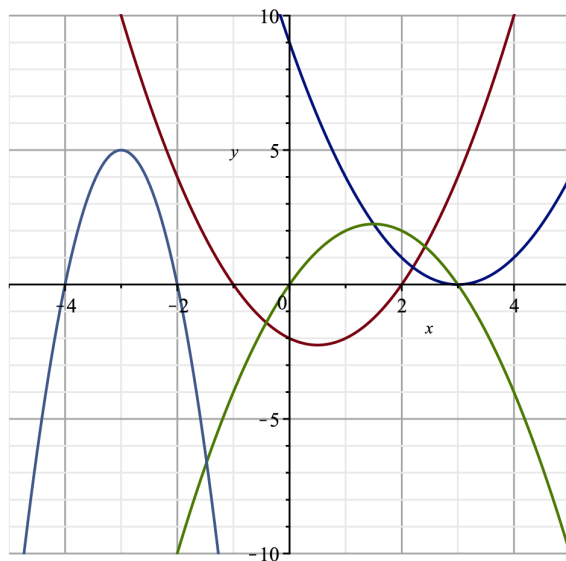
    iv. $y = -(x-2)(x+1)$

    v. $y = (x+2)^2$

    vi. $y = x^2 - 2$

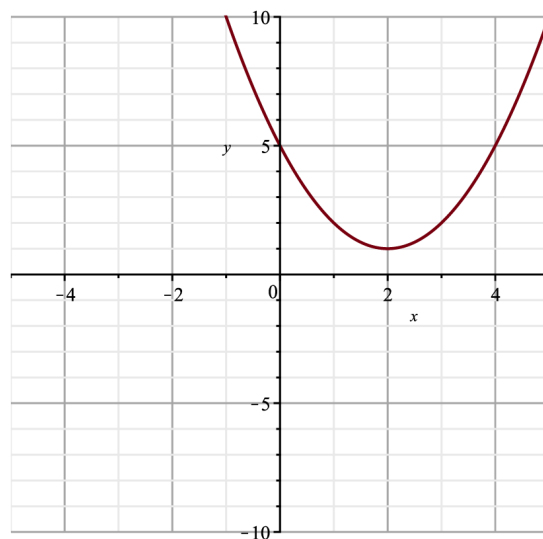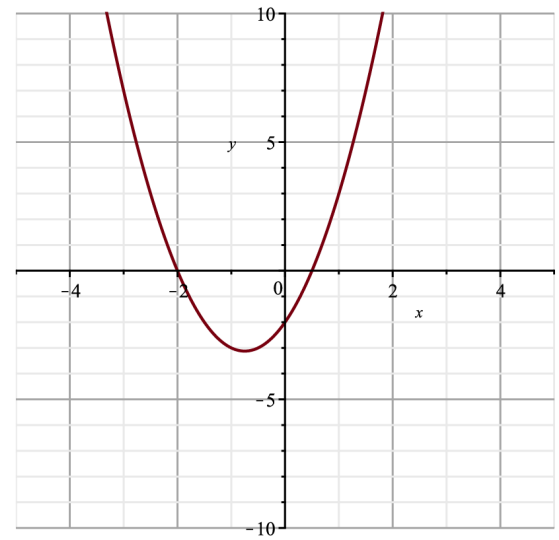vii. $y = 0.25(x+2)(x-1)$

viii. $y = -0.33(x+3)^2+1$

d.

e.

   i.

x	y	1st difference	2nd difference
0	5		
1	2	−3	
2	1	−1	2
3	2	1	2
4	5	3	2
5	10	5	2

ii.

x	y	1st difference	2nd difference
−3	7		
−2	0	7	
−1	−3	3	4
0	−2	−1	4
1	3	5	4
2	12	9	4

f. An important part of parabolas are that they are symmetrical. The line of symmetry is centred on the vertex of the parabola. This means that if you were to reflect the parabola along this line of symmetry you would get exactly the same graph.

This is really useful because it means that each point on either side of the vertex is the same distance from the line of symmetry. This can come in handy when only one root is seen on the graph because the other root will be an equal distance away from the centre of the parabola.

3. Understanding Exponential Equations

a. An exponential equation is an equation that has some number (the base) to the power of a variable. Some examples are 2^3, where 2 is the base and x is the exponent, and 3^x, where 3 is the base and x is the exponent. The general form for an exponential equation at this level is $y = k × a(x−b) + c$, where a is the base, k is the scale, b is the horizontal shift, and c is the vertical shift.

b. Exponential equations are good for modelling the growth of populations and other things that grow faster as they get larger. For example, we can use exponential equations to model how much area some bacteria takes up, the

growth of a population of humans or animals, the size of a fast-growing plant, or the amount of money you have if you were to double, triple, or quadruple it on a regular basis.

c. Some features we would expect to see on the graph would be a y-intercept, horizontal asymptote.
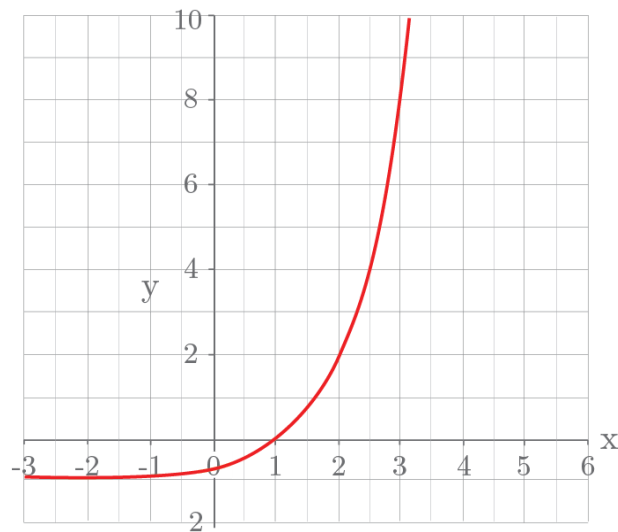
d.

    i. $y = 2^x$

    ii. $y = 4^x$

    iii. $y = 3^x$
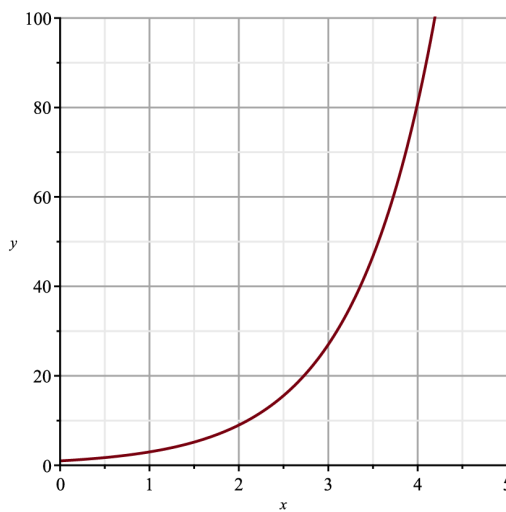
    iv. $y = 3^x + 2$

    v. $y = 2^{(x+1)}$

Change Note: In previous copies of the workbook, the graph for the following answer (d. vi.) had been printed incorrectly. Here is the correct graph:

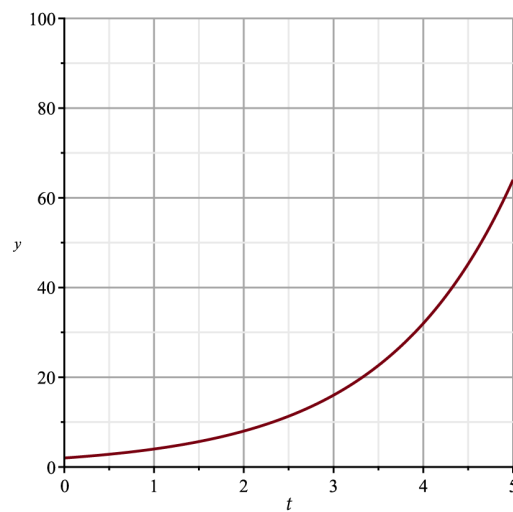   vi. $y = 3^{(x-1)} - 1$

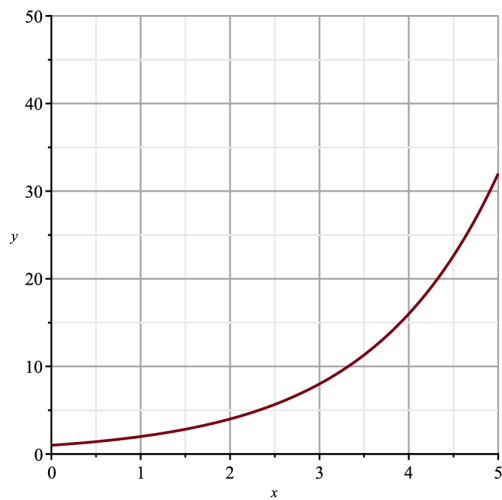e.

i. $y = 3^x$

ii. $a = 2^{t+1}$

f.   When we're considering the growth of something over time, we can think of the area as being the y value, and the time being the t value (or sometimes x). In this case, we want to know what the area is when no time at all has passed. That means we want to know what y was when t=0. We can find this either by looking at the y-intercept on the graph, looking at our table to see what y was when t=0, or by substituting t=0 into our equation to calculate y. Any of these methods will give an area of 2m^2, meaning the area when the plant was first measured was 2m^2.

g. When the base is less than 1, the shape of the graph is essentially reflected on the y-axis. The reason for this is a bit beyond level 1 but is very important next year and gives a good link between what you'll have to do in the next section to reflect graphs.

The value of y gets smaller when the power is greater than 0, because you are multiplying a number smaller than 1 by itself which gives an even smaller number. When you are taking it to a negative exponent you are multiplying a number less than 1 by a larger and larger number. So the value of y increases rapidly for powers less than 0.
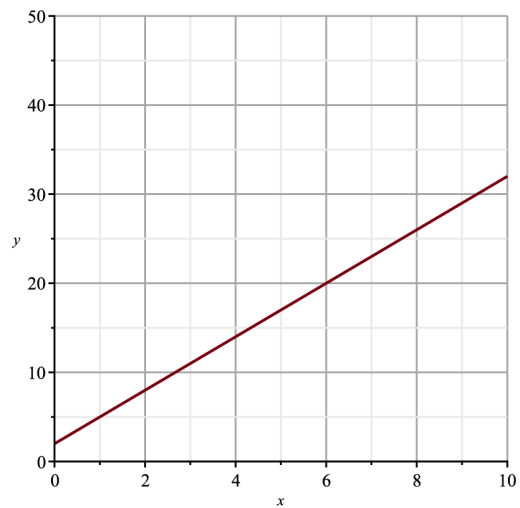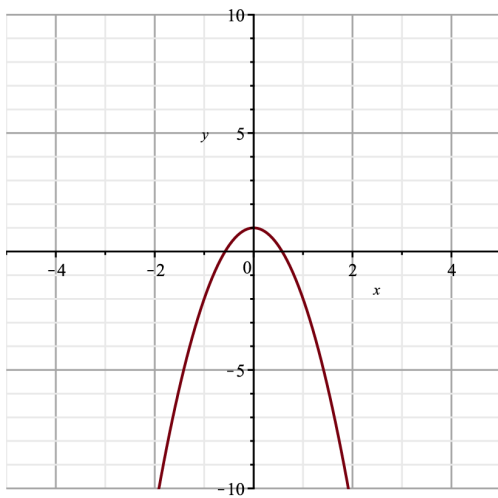
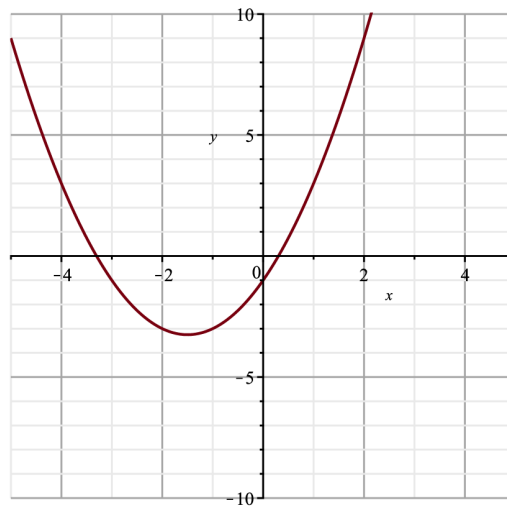# 4. Putting It All Together

a.

i. $y = x^2$

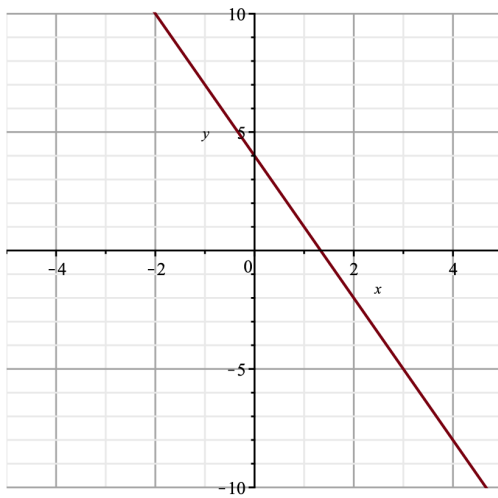ii. $y = 3x + 2$

iii. $y = -3x^2 + 1$

iv. $x^2 + 3x - 1$

Level 1 Maths | Tables, Equations and Graphs |
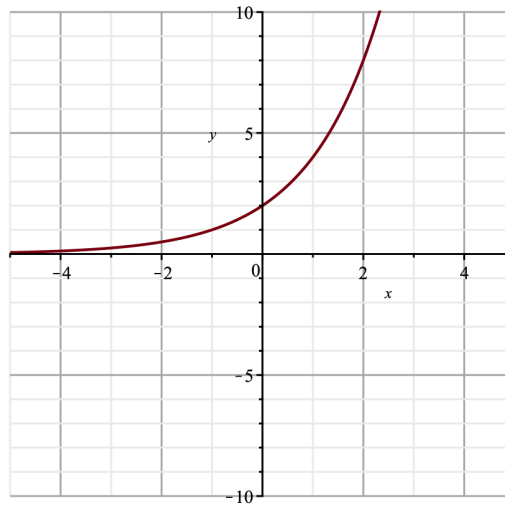
b.

i. $y = -3x + 4$

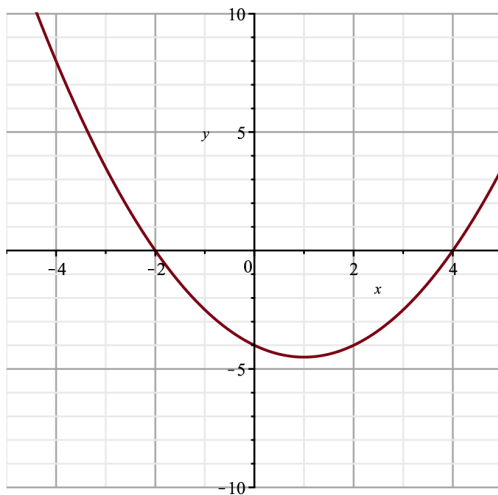

Gradient = -3

y-intercept = 4

x-intercept = $\frac{4}{3}$

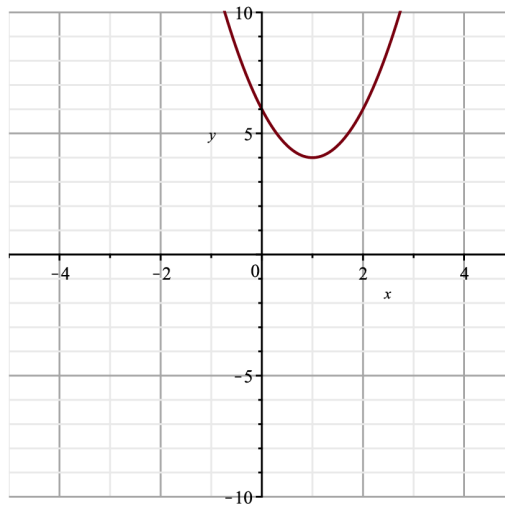ii. $y = 2^{x+1}$

y-intercept = 2

iii. $y = 0.5(x - 4)(x + 2)$

x-intercepts = -2 and 4

y-intercept = -4

Vertex = $(1, -4.5)$

iv) $y = 2(x - 1)^2 + 4$

No x-intercepts

y-intercept = 6

Vertex = $(1, 4)$

c. The general idea here is to figure out whether we need to use the vertex form or the intercepts form, based on what information we can easily read off the graph. Write out the equation with the information you know (either the vertex or the intercept). There will still be a k in the graph, so to find what that is, choose another point and substitute it in to solve for k. Don't forget to rewrite your equation once you know what k is!

    i. $y = 0.5\,(x + 2)(x - 4)$

    ii. $y = 2\,(x - 1)^2 + 4$

    iii. $y = 2(x + 1)^2 - 3$

    iv. $y = -0.5(x + 1)(x - 3)$

Section One – Part Two

1. Vertical Shift of Graphs

a. For linear, quadratic and exponential equations we can just add or subtract a constant from the original equation. The reason this works is because we are adding or subtracting this constant from the value of y for a given x-value. For example, to shift a graph up by 1 unit, we add 1, to shift it down 3 units we subtract 3.

b.
    i.   Add 2 to each y value to give $y = 2x + 2$. This changes the y-intercept to be at $y = 2$.

    ii.   This gives the equation $y = x^2 - 1$. We could factorise to give $y = (x + 1)(x - 1)$ which means the roots are now $x = 1$ and -1.

iii. This gives $y = 2^x + 1$ which changes y-intercept to be 2 and the asymptote is now $y = 1$.

iv. This gives $y = -\dfrac{1}{3x} + 4 - 2$ which simplifies to $y = -\dfrac{1}{3x} + 2$. The y-intercept is now 2.

v. This gives $y = x^2 + 3x$ which we can factorise to give a new equation $y = x(x + 3)$. This means our new roots are $x = 0$ and $x = -3$.

vi. This gives the equation $y = (x - 1)(x + 2) - 4$. We expand out the brackets and simplify to get $y = x^2 + x - 6$. By factorising again so we can find the roots, we get $y = (x + 3)(x - 2)$ which has roots $x = -3$ and $x = 2$.

c.
i. Shifting the graph vertically changes the y-intercept by however much is added or subtracted. So if the vertical shift is a, then the new y-intercept is $c + a$. Remember that a can be negative too, which is the shift down.

ii. Similarly to above, shifting the graph vertically means that for any particular value of x, the value of y will increase or decrease by the shift. So if the vertical shift is a, then the new y value will be $y + a$.

2. Horizontal Shift of Graphs

a. To do this, we need to add or subtract some amount for every x value in the given expression. If we are adding to the x-value, this has the effect of shifting the graph to the left. Alternatively, we shift the graph further to the right by subtracting from the x term.

b. For all of these we subtract from the x-values to shift to the right and add to shift to the left, we have to be very careful to add or subtract from the x values only, to do this we use brackets and then simplify where appropriate.

i. $y = 3(x+1) + 1$

So by expanding we get $y = 3x + 4$, which means the y-intercept has changed to 4.

ii. $y = (x - 2)^2$ which makes the new x-intercept 2.

iii. $y = -(x - 2)(x + 2)$.

So our x-intercepts have changed to $x = 2$, $x = -2$.
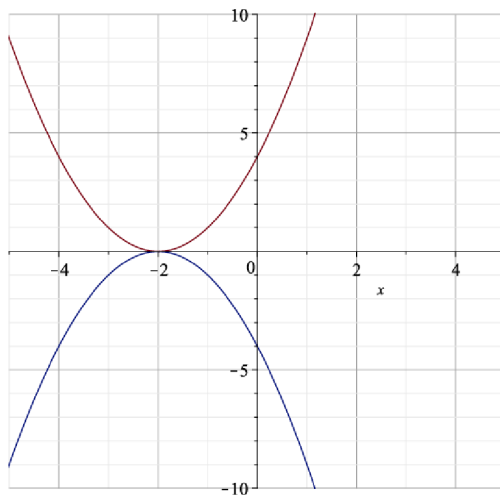
iv. $y = 2^{(x+3)}$

v. $y = 2(x+2)^2 - 8$. It is okay to leave it in this form because it is vertex form. The vertex is at $(-2, -8)$.

vi. We get $x - 3 = 2$, which is $x = 5$. This gives us an idea of why we add to x-values to move left, and subtract to move right.
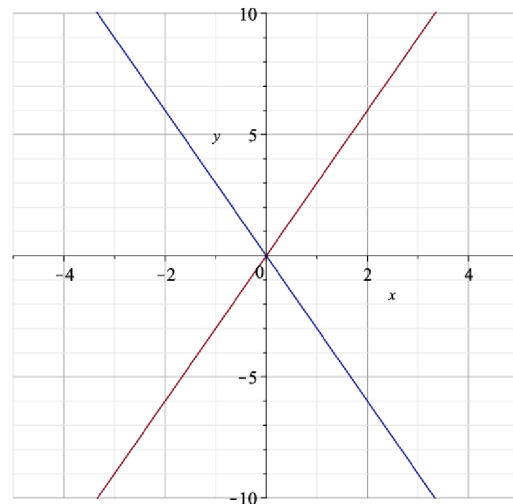
3. Reflecting Graphs

a. For most of these problems, except for question vi, we can multiply everything by -1 to reflect the graph on the x-axis.
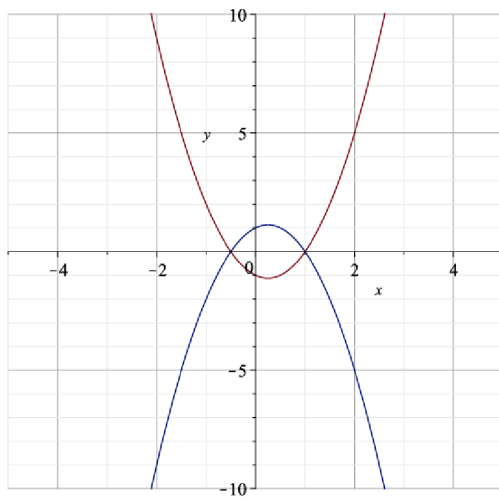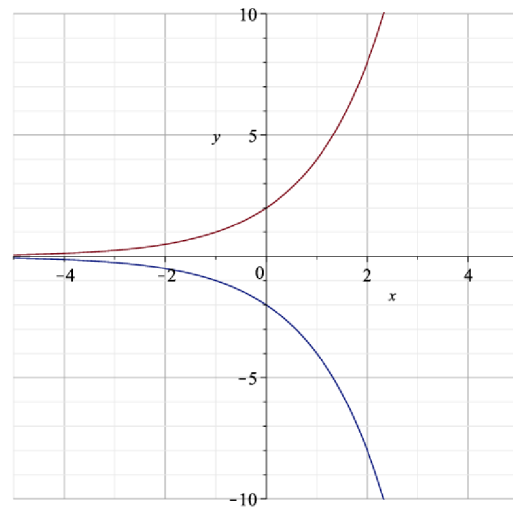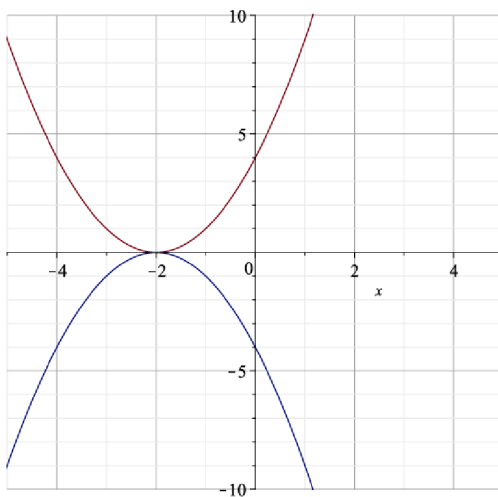
i. $y = - (x + 2)^2$
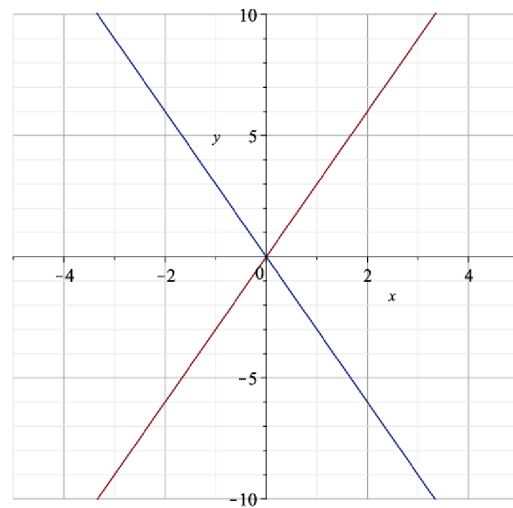
ii. $y = -3x$

iii. $y = -(2x^2 - x + 1)$

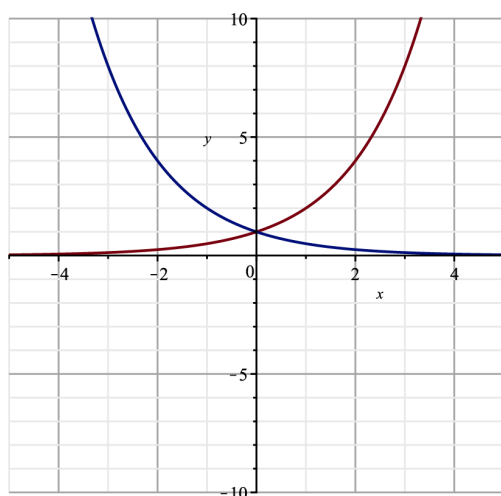iv. $y = -(2^{x+1})$

v. $y = -\dfrac{1}{4}x + 4$

vi. This one is a little bit harder. First, you reflect the graph like normal, multiplying everything in the equation by negative 1. To get it to reflect at the vertex, you then have to shift it up by 8 units. This gives the new equation:
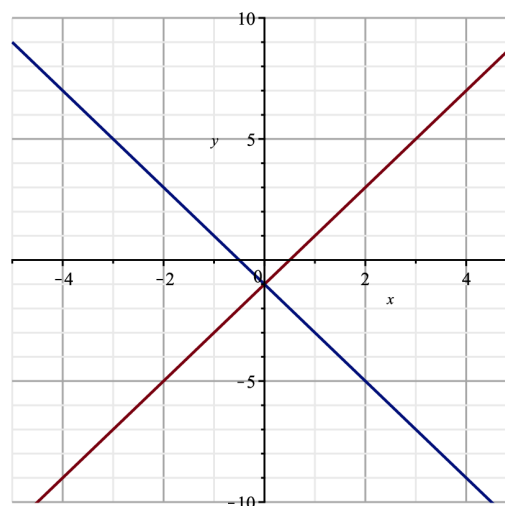$y = -(x - 1)^2 + 4$.

b. For all of these we have to multiply the x value by -1. This gives a positive number for every negative x value, and negative number for every positive x value, reflecting the graph! You can simplify some of these if you want but it's not necessary.
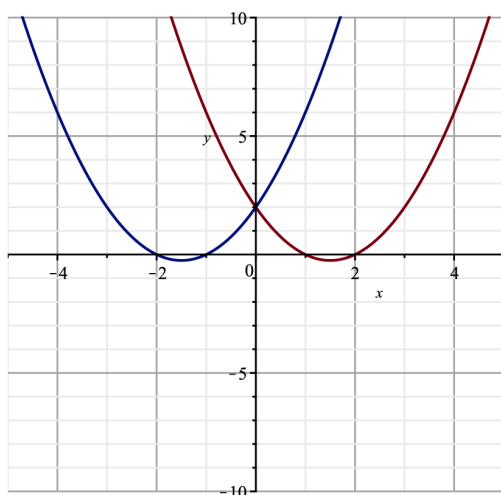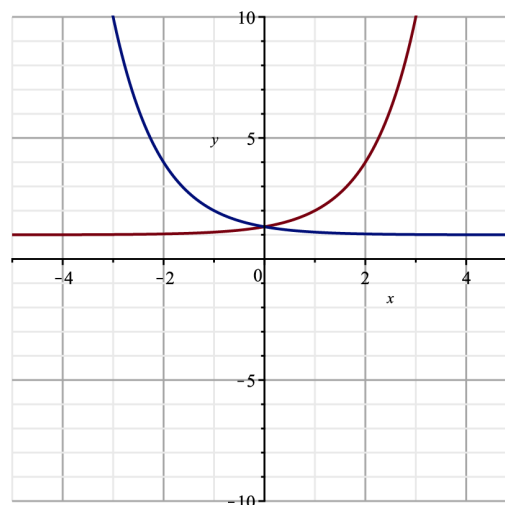
i. $y = 2^{-x}$

ii. $y = 2(-x) - 1$

iii. $y = (-x)^2 - 3(-x) + 2$

iv. $y = 3^{-x-1} + 1$

4. Reflecting Graphs

a. Linear: the higher the scale factor, the higher the gradient/slope.

Parabola: the higher k-value/scale factor, the greater y increases for every 1 unit increase or decrease of x, so the y-value of the vertex changes. However, the x-intercepts remain the same if the graph is in intercept form. If it is in vertex form, the k factor is going to change all of the y values where $(x - a)$ is not zero, which means the vertex will remain the same but the intercepts will change. Visually, the larger the scale factor is, the taller and thinner the parabola is. The smaller it is, the wider and fatter the parabola is.

Exponential: the higher k/scale factor, the greater y increases for every 1 unit increase or decrease of x. However, it still crosses the y-axis at the same y-intercept, and the asymptote remains the same.

b. To find k, you need to have both a value for x and a value for y. For example, if you had the equation $y = (x - 2)(x - 4)$ and you had the point $(3, -2)$:

$$-2 = k\,(3 - 2)(3 - 4)$$
$$-2 = k\,(1)(-1)$$
$$-2 = -k$$
$$\frac{-2}{-1} = k$$
$$2 = k$$

So the equation is $y = 2(x - 2)(x - 4)$.

c.

    i. $y = k(x - 3)2 - 30$

         $k = 4$ so $y = 4(x - 3)2 - 30$

    ii. $y = k(x - 3)(x + 7)$

         $k = -0.85$ so $y = -0.85(x - 3)(x + 7)$

    iii. $y = k \times 4^x$. We substitute in a point, in this case either $x = 1$ or $x = 2$ and we find that k is 0.5 so the equation is $y = 0.5 \times 4^x$.

iv. From the intercepts we get

y = kx(x - 6). We substitute in a clean point which appears to be at (-4, 80) or (9,80) which gives k = 2 so y = 2x(x - 6)

d.

i. Scaling a quadratic equation in vertex form is going to change the x-intercepts, or the roots. Scaling a quadratic equation in intercept/factorised form is going to change the vertex of the graph. Think about why this is.

ii. Scaling an exponential graph is going to change the y-intercepts and will shift all other points on the graph either up or down depending on the value of k. If k is less than 1 then all points will be less than the original and if it is greater all points will be greater. The value of the intercept will become (0,k).

5. Putting it All Together

a. For all of these problems, we have to shift graphs left and right (by adding and subtracting from their x-values) and shift them up or down (by adding or subtracting from the equation). Again, you can simplify if you want, but we would recommend not simplifying.

i. $y = (x + 2) + 3$

ii. $y = -(x)^2 - 2$

iii. $y = 2(x - 1)^2 + 3(x - 1) - 6$
Simplified, this is: $y = 2x^2 - x - 7$

iv. $y = 3^{(x+1)} + 2$

v. $y = 2^{(x-1)} + 2$

vi. $y = -0.5(x - 2)(x - 4) + 2$

b. This equation has solutions a x = -3 and 2, so if we shift the solutions 2 to the right that will give solutions at x = -1 and 4. We could then use these solutions to write our new equation in factorised form, which would be:

$y = (x + 1)(x - 4)$

Section Two - Part One

Understanding Contextual Problems

1. Discrete data is usually counted and can only take certain values. For example, people. You can't have 2.5 people, only 2 people or 3 people. Therefore, as it can only have certain values it is discrete.

    Continuous data is usually measured and can take any value. For example, height. You can be 1.76m tall or 1.8215m tall. Height can be any value and so therefore is continuous data.

2.
   a.
   i. The dependent variable is the cost of the scooter, since it depends on the whole number of minutes that the scooter is rented for. The time would be on the x-axis and the cost would be on the y-axis.

   ii. The independent variable, time in minutes, is continuous. However, the cost is not continuous as it only changes after each minute has passed. This means that the cost of 2 minutes will be the same for 2 minutes

and 30 seconds, or 2 minutes and 59 seconds, therefore the cost is a discrete variable.

    iii.   The cost of renting the scooter for each minute passing increases by the same amount, so the increase in cost would be linear.

b.

    i.   The dependent variable is the area of the fungus, since it depends on the number of days it has been growing for which is the independent variable. The area will be on the y-axis and time on the x-axis.

    ii.   The dependent variable is going to be continuous, because the area of the fungus is measured constantly by the camera and can take on any value (i.e. the area could be $1.0005m^2$ or $3.14159m^2$ for a given time). The independent variable is also going to be continuous because we can have any amount of time passing.

    iii.   This is a classic exponential situation. The more fungus there is, the faster it can grow, which means it grows exponentially.
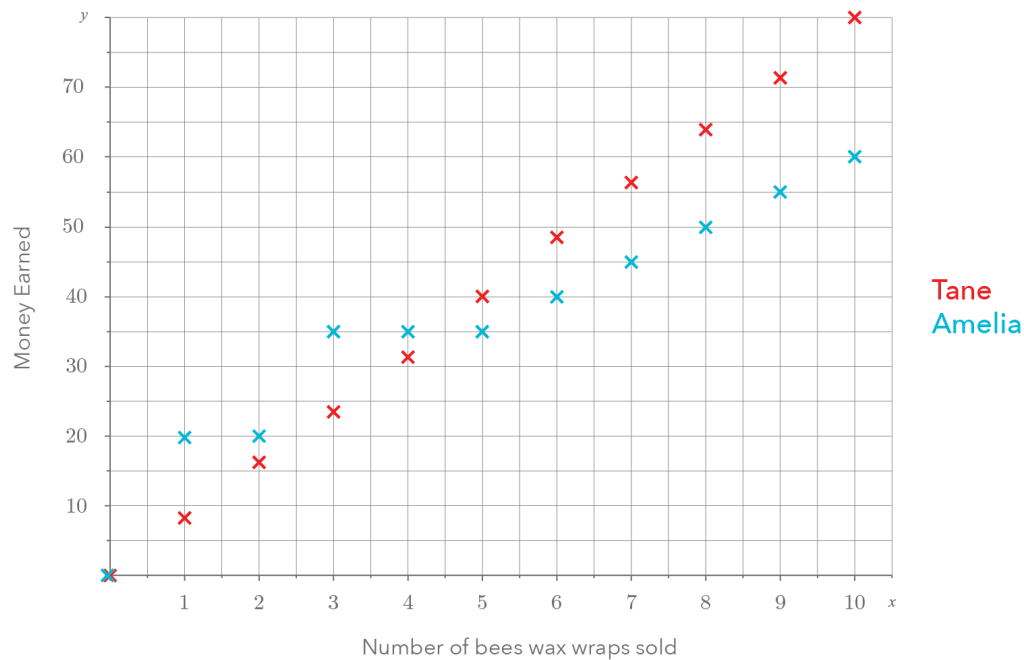
c.

    i.   The area of the pond weed is the dependent variable, since it depends on the day that the area is measured. The area will be on the y-axis and the time will be on the x-axis.

    ii.   This is a bit tricky, in reality, the variables are continuous, any amount of time can pass and the area of the pond weed can be anything. However, the area is only measured at the end of each day, so we just have one area for each day, so we would probably plot discrete points on the graph!

    iii.   This is another classic exponential relationship because the more pond weed there is, the faster it can grow.

d.

    i.   The area is the dependent variable, it depends on the length of the sides of the rectangle. The area would be on the y-axis and the value of x (and therefore the length of the sides) would be on the x-axis.

    ii.   Both of these variables are continuous, since they can have any value (greater than zero). We could easily imagine a rectangle with sides of any length.

    iii.   This is a bit tricky but for any length of the sides, the area will be the sides multiplied together, so we would have a quadratic relationship. E.g. if the sides were x and x+1 the area would be A = x(x+1) which is A = x^2+x.

e.

    i.   The number of students who have the viral infection is the dependent variable. It depends on the number of days which is the independent variable.

    ii.   The number of the students with the infection is going to be discrete. We can't have half of a person who has the infection, only whole numbers of infected people. The days that have passed could be discrete or continuous, but are likely to be continuous.

    iii.   This would definitely be an exponential equation to begin with, because the number of people who can be infected depends on the number of people who have the infection. It may be modelled by some other kind of equation after a certain amount of time has passed.

Section Two - Part Two

1.

Money Earned

Number of bees wax wraps sold

Tane
Amelia

a.
- i. It doesn't make sense to sell half of a wrap, or 2.492 wraps. Therefore the variables must be discrete. Because the variables are discrete, we will only plot one point for each full wrap, making sure to not draw a line through them.

- ii. It's possible that they didn't sell any wraps, in which case they would have sold 0 wraps (and made $0). The maximum they can make for Tane's model is $80 for 10 wraps. For Amelia's model, if they sell 10 wraps they make $35 + $25 = $60. So we should probably draw the x-axis between 0 and 10 and the y-axis between 0 and 100.

b.
- i. $20 for two wraps.

- ii. $35 for selling 5 wraps.
- iii. Two possible answers, one where they sell 2 wraps and then 3 more for a total of $20 + 35$ = $55. So this is $11 per wrap.

c.

i.   Tane's model makes $40 selling 5 wraps, whereas Amelia's model makes $35. Therefore to make the most money we'd recommend Tane's model.
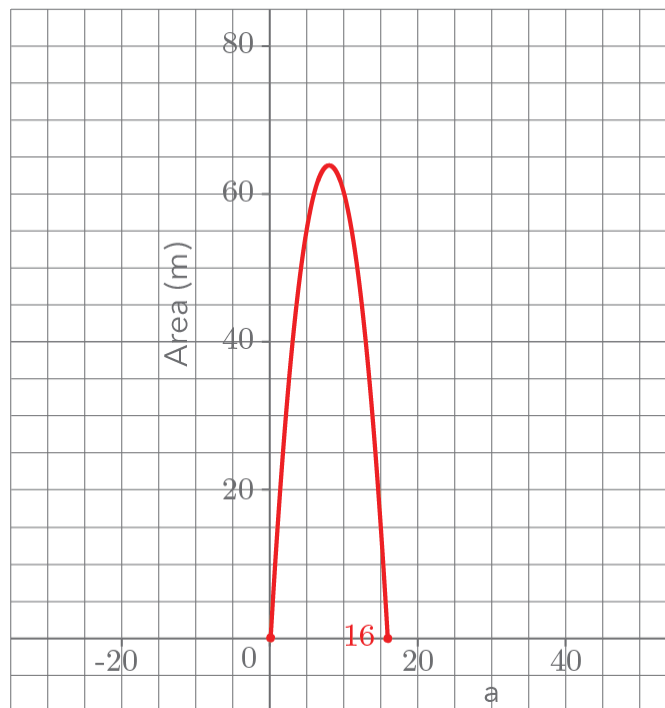
2.

a.

i.   We know that the original plank is 16m long, so if we cut one plank into a plank that is a metres long the other is 16 - a metres long. We could call this other plank b, which would mean that it's b = 16 - a.

ii. This would be the length of a multiplied by the length of b, which is Area = ab. We could write in terms of a by substituting in our equation from i., giving us the equation Area = a(16−a). To help us understand this equation better, it helps to use a table to work out values of a and b and the corresponding area and then draw it on a graph. You could also use the graph to work out the equation, instead of using Area = ab.

iii.

Plank 1: a	Plank 2: b = 16 − a	Area: A = a(16 − a)
1	15	15
2	14	28
3	13	39
4	12	48
5	11	55
6	10	60
7	9	63
8	8	64
9	7	63
10	6	60

iv.

v. Using the graph, we see that there are two intercepts at 0 and 16 giving the equation:
A = ka(16 − a), if we substitute in any point except for the intercept we see that k is equal to 1 so the final equation is A = a (16 − a).

b.

i. The vertex of the equation is at the point (8, 64), where the area is 64, so $64m^2$ is the maximum possible area of the rectangle/table.

ii. The length of both planks is 8m. Mathematically speaking this is the length of the original plank divided by 2, which tells us that the square is actually the largest possible area.

c.

i. If the original plank is n metres long, then when we cut it into 2 we'll get one plank which is a metres long and another which is n − a meters long.

ii. Given that we've cut our original planks into planks of size a and n - a, then we can calculate the area with the equation Area = a(n - a).

iii. Looking at our equation, we can see our roots/x-intercepts will be at a = 0 and a = n. The x coordinate of the vertex will be at a = $\frac{n}{2}$, because parabolas are symmetric. Since the maximum area will be at the vertex, we can substitute a = $\frac{n}{2}$ into our equation to get the maximum area.

iv. If two planks n metres long are cut into planks of size a and n - a, then the maximum area will be when all of the planks are $\frac{n}{2}$ metres long. This means that our maximum area will be calculated by Area = $\frac{n}{2} \times \frac{n}{2}$, which would give us an area of $\frac{n^2}{4}$ m².

3.
a. We could draw a table:

Layer (n)	Lines per layer (l)
0	1
1	2
2	4
3	8

l = 2^n where l is the number of lines in the layer and n is the layer number, starting from 0.

b. The equation tells us that this would be l = 2^{10} which is 1024!

c. Well, we could again draw a table but in this case, we can just imagine that we are starting from the second layer, which means that we are going to be

adding one to the power for each layer or $I = 2^{n+1}$. You may find it useful to write out a table to see how this equation gives us what we want.

Section Three - Practice Exam
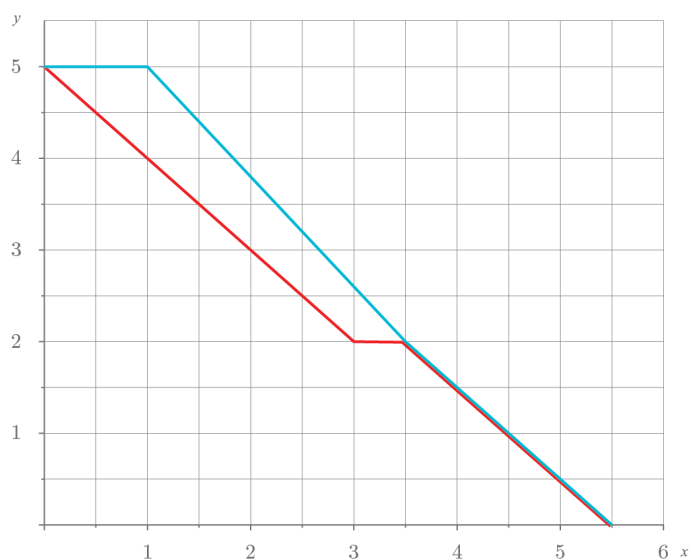
Question 1

a. $y = -\frac{6}{9}x + 6$

A for correct equation

b. Their average speed is the change in their height from sea level divided by the time that it took them. The change in their height is -5km (because they are going down), so their average speed is $-\frac{5}{3}$km/hr.

A for correct gradient, M for reference to speed with correct direction.
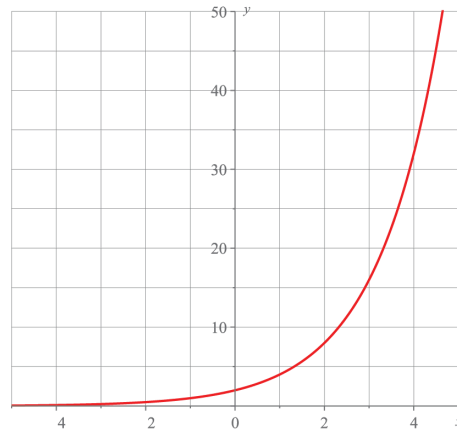
c.

i.

Climber 1
Climber 2

ii. $y = -x + 5$

iii. For the first hour, climber 2 doesn't move, so we can represent their distance as $y = 5$. For the next part of their journey, they descend at a rate of 1.2km per hour for 3 hours, giving an equation of $y = 1.2x + 6.2$. Then after the first hour, the climber is descending at a speed of 1.5km per hour, for 3 hours at a constant rate. The first climber descends the cliff at a rate of 1km per hour for the first 3 hours, at a constant rate. When the first climber is taking a rest, the equation is $y = 2$. Then after the 30min break and when climber two meets climber 1, they descend at a speed of 1 km per hour. The equation for both climbers is given by $y = -x + 5.5$.

Climber 1 covered the first 3km faster than climber 2 because they started an hour sooner. However, climber 2 was the faster climber, moving at 1.2km/h rather than 1km/h like climber 1. They finished the last 2km of their journey at a pace of 1km/h, meaning that climber 2 had to slow down and climber 1 stayed the same pace.

Question 2

a.

b. $y = 2^{x+2} + 4$

c.

    i.

    ii.   The equation for the situation is $y = 4^{n-3}$, so this needs to be larger than 1024. 4^5 is equal to 1024 so we want an n so that n-3 > 5. Rearranging

this, we see that if we have n > 8, then our y will be larger than 1024. The length of the molecule is going to need to be over 8 molecules long.

iii. This is a little bit tricky, but it's helpful to think about it in this way. A length of DNA can have 4 combinations per molecule which for n molecules is 4n combinations. If one is fixed, this is 4^{n-1}, if two are fixed it has 4^{n-2} combinations, and so on. So, it is reasonable (and accurate) to say that for a molecule of length n, where c of these molecules are fixed, that the equation is going to be 4^{n-c} possible combinations.

Question 3

a.

i.

x, Bonnie's number	x − 6 = A	x + 2 = B	y = AB
0	−6	2	−12
1	−5	3	−15
2	−4	4	−16
3	−3	5	−15
4	−2	6	−12
5	−1	7	−7
6	0	8	0

b.

i. $y = (x - 6)(x + 2)$

The graph tells us that the answer to Bonnie's puzzle is that her answer can either by 8 or - 4.

M for correct equation and one positive answer. E for both solutions, justified that it can be negative with reference to the graph.

ii. Your answer should include something about the fact that this wouldn't be possible. We can see that on the graph, which represents all possible numbers that Bonnie is thinking of, that the answer −20 doesn't have a corresponding x-value. This means that there is no number for x that gives −20 as a solution.

E for mentioning that the curve represents all possible solutions, reference to no solution.

c.

i. Todd's number is 11.

A for getting correct number.

ii. We can use the equation $3x - 24 = 9$ to find Todd's number. If we multiplied by 3 instead of by 4, then the equation would change to:

4x − 24 = 9.