Workbook Answers
Level 1 Maths
Geometric Reasoning

Section One

1. Angles on a Line

a.

i. The angle is 360°, because it is a circle.

ii. No, the angle is still 360°, which links to the rule for angles at a point.

iii. The angles A, B, C, D, E and F add to 360°. The rule is angles at a point add to 360°.

b.

i. Because we have cut the circle in half, the angle around the centre is 180°, half of 360°.

ii. The angle around the centre is still 180°.

iii. 180°. The rule is angles on a line add to 180°.

c.

i. These angles all lie on the straight line AOD, so adding them gives 180°. This comes from the fact that we can get a straight line from halving a semicircle.

ii. Subtract the 90° angle from the total 180°.

COA = 180° – 90°

COA = 90°

iii. We know angle COA is 90°, so 90° – 25°= 65°. So the angle BOA is 65°.

d.

i. All of the angles meet at a point, so they will add to 360°.

ii. BOC is the only angle we don't know here, which means that we can write:

$\angle BOC = 360° - \angle AOE - \angle EOD - \angle DOC - \angle AOB$

Using numbers we get:

$\angle BOC = 360° - 40° - 115° - 55° - 90°$

$\angle BOC = 60°$.

2. Basic Triangle Skills

a.

i. Angles in a triangle add to 180°.

ii. Because the angles add to 180° we can write:

$60° + 45° + x = 180°$

Rearrange to get:

$x = 180° - 60° - 45°$

$x = 75°$.

b.

i. Right angled: 1, 3.

Isosceles (at least 2 sides are the same): 2, 3, 4.

Equilateral (all sides are the same): 4.

ii. 1, 3 because they have right angles, and we can only use Pythagoras and trigonometry with right-angled triangles.

iii. Interior angles of triangles always add to 180°.

iv. The other two angles are both 60° because it is an equilateral triangle, meaning all the sides are equal and all the angles are equal.

v. One of the angles is 90° (as depicted by the right angle symbol).

Because angles in a triangle add to 180°, we work out the other angle by calculating:

$180° - 90° - 45° = 45°$.

vi. One of the angles is 90°.

The other is $180° - 90° - 15° = 75°$.

c.

i. Angles in a triangle add to 180°, so we calculate

$CAB = 180° - 25° - 35°$

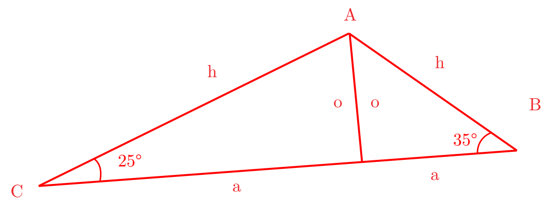
$CAB = 120°$

ii. Pythagoras and trigonometry only work for triangles that have a right angle in them, so since this isn't a right-angled triangle we can't use them.

iii./iv. We need to split this triangle into two right-angled triangles to work with. The original triangle was drawn out of scale to throw you off (which might happen in your exam too!) so we have redrawn it more accurately here.

We chose to draw the new line from vertex A to meet the line BC at a right angle, making two right-angled triangles. You may have drawn it differently, although you may find the next couple of steps more challenging if so since we don't know the lengths of all of the sides.

We have labelled their sides as adjacent (a), opposite (o), and hypotenuse (h), using ∠ACB and ∠ABC as the angles which the sides are opposite/adjacent to for their respective triangles.



v. We have the hypotenuse of each of the right-angled triangles, so we use $\sin(\theta) = \frac{o}{h}$.

vi. Now we calculate the opposite lengths using 25° and 35° and the length AC and AB respectively.

For the left-hand triangle, we have:

$\sin(25°) = \frac{o}{4}$

Rearrange to get:

$o = 4 \times \sin(25°)$

$o = 1.69$m (2dp)

For the right-hand triangle,

$\sin(35°) = \frac{o}{2.59}$

Rearrange to get:

$o = 2.59 \times \sin(35°)$

$o = 1.69$m (2dp)

They are both the same, which is what we were hoping for!

3. Angles in Parallel Line

a.

i. Vertically opposite angles are equal so ∠DOC = ∠AOB.

So ∠DOC = 39°.

ii. Corresponding angles on parallel lines are equal so ∠ABC = ∠GFC

So ∠ABC = 30°.

iii. Co-interior angles on parallel lines add to 180°, so

$\angle EFC = 180° - \angle FDC$

$\angle EFC = 180° - 117°$

$\angle EFC = 63°$

iv. Alternate angles on parallel lines are equal so $\angle BEF = \angle CBE$.

So $\angle BEF = 40°$.

a.

i. Corresponding to 60°.

ii. 60° (since corresponding angles on parallel lines are equal).

iii. Co-interior to 60°

iv. Co-interior angles on parallel lines add to 180°, so

$y = 180° - 60°$

$y = 120°$

v. Angles on a line always add to 180°.

vi. $z = 180° - 60° - 90°$

$z = 30°$

vii. The angle that is vertically opposite to x is co-interior to z + 90°. Since vertically opposite angles are equal, this angle is equal to x and so is 60°. You could also have worked out this angle using the straight line rule with y.

viii. Co-interior angles on parallel lines add to 180°, so

$z + 90° = 180° - 60°$

$z + 90° = 120°$

$z = 30°$

4. Angles in Circles

a.

i. This is called the diameter of the circle. Any line that goes from circumference of the circle through the centre to the circumference is the diameter.

ii. This is called the radius of the circle. A radius is any line that goes from the centre of the circle to a point on the circumference.

iii. Line AD is a chord. A chord is a line that goes from one point on the circumference to another point on the circumference without going through the centre.

iv. A line that touches the circumference only once is called a tangent.

b.

i. Any angle which is contained within a semicircle, made from two lines which start at either end of the diameter and meet somewhere on the circumference, is a right angle. Therefore this angle is 90°.

ii. The answer is that any angle at the centre is twice the angle at the circumference if they are on the same arc. So $y = \frac{1}{2}x$

iii. Because the line goes from the centre to the circumference it is the radius. The angle between the radius and a tangent is 90°, so x = 90°.

iv. Opposite angles in a cyclic quadrilateral add to 180°, therefore a + c = 180° and b + d = 180°.

v. Angles standing on the same arc are equal. So x = y.

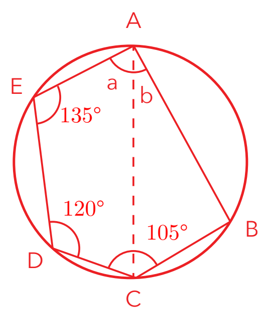
c.

i. Using the fact that the radius is perpendicular to the tangent, x = 90°.

ii. Now that we have two of the angles in the triangle we can calculate the third.

∠COD = 180° − 90° − 30° = 60°. So y = 60°.

Note for question d: in previous versions of the Workbook, angle C was incorrectly labelled as 135°, and has been updated to 105°. We will include both answers where relevant, but we recommend you go back and revise your answer using the updated diagram below.



d.

i. Diameter

ii. Cyclic quadrilateral

iii. Opposite angles in a cyclic quadrilateral add to 180°. So

a + 120° = 180°

a = 180° − 120°

a = 60°

iv. 135° + ∠ACD = 180°, so

∠ACD = 45°

v. ABC is a right-angled triangle. The rule is that a triangle in a semicircle will always be a right-angled triangle.

vi. Angles in a semicircle are right-angled, so

$\angle ABC = 90°$

vii. $\angle ACB = 105° - 45°$

$\angle ACB = 60°$

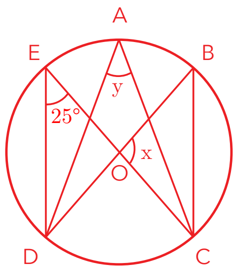
Old answer: Angle ACB = 135°– 75°= 60°

viii. b = 180° – 90° – 60° (angles in a triangle)

b = 30°

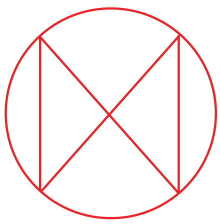
Note for question e: same drill again, in an older version of the Workbook this diagram had been incorrectly labelled (look, we're working on it haha *sweats nervously*). In this case we will provide the updated question and diagram for you to revise your answer:

e. New question: The centre of this circle is O, and all other points are on the circumference. The goal is to find angles x and y.

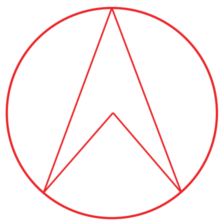


i. New question: Identify at least two rules you could apply to the angles in this circle.

Answer:



Rule: Angles on the same arc



Rule: Angle at centre = twice angle at circumference

ii. New question: Use one of your rules to find the size, y, of angle DAC.

Answer: y = 25° (angles on same arc).

iii. New question: Now we want to find x, the size of angle BOC. Firstly, calculate the angle of DOC, writing down the rule you use. Then work out the size of angle BOC. Again, write down the rule you use.

Answer: Angle DOC is twice the size of DAC (angle at centre = twice angle at circumference). Therefore

$\angle DOC = 2 \times 25° = 50°$.

Then:

$\angle BOC = 180° - 50°$ (angles on a straight line)

$\angle BOC = 130°$

So x = 130°.

5. Interior and Exterior Angles in Polygons

a.

i. A polygon is a shape made up of straight lines (these are called edges). A regular polygon is made up of straight lines of the same length. Also, all of the internal angles are the same size as each other.

ii. We end up facing the same direction as before, so we've turned 360°. This will be true for any polygon.

iii. $360°/3 = 120°$

iv. Sum of angles in a triangle = 180°

v. $120° +$ interior angle $= 180°$

This means that:

Interior angle $= 180° - 120°$

Interior angle $= 60°$

vi. $60° \times 3 = 180°$, so yes it is the same - which it should be!

vii. n is the number of edges (or angles) in a polygon. So this polygon has $n = 3$.

viii. Sum of interior angles $= 180° \times (n - 2)$

Sum of interior angles $= 180° \times (3 - 2)$

Sum of interior angles $= 180°$

b.

i. For any/all polygons, the sum of the exterior angles is 360°

ii. Square: $360°/4 = 90°$

Pentagon = $360°/5 = 72°$

Hexagon: $360°/6 = 60°$

iii. Square: $180° \times (4 - 2) = 360°$

Pentagon = $180° \times (5 - 2) = 540°$

Hexagon: $180° \times (6 - 2) = 720°$

iv. Square: $360°/4 = 90°$

Pentagon: $540°/5 = 108°$

Hexagon: $720°/6 = 120°$

c.

i. Single exterior angle = $360°/n$

ii. Interior angle = $180 - 360°/n$

iii. Sum of interior angles $= 180n - 360 = 180(n - 2)$

d.

i. Sum of exterior angles of a polygon is 360°, and all angles of a regular polygon are equal. So

$x = {}^{360°}\!/_{5}$

$x = 72°$

Note for question d - ii, d - iii: ok last one, I promise. Older versions of this workbook ask you to find x in these questions, but it should be asking you to find y. We will include the updated question with the answers here, and we encourage you to go back and revise your answer based on this. You don't have to but, like, it'd be so cool if you did.

ii. New question:

The interior angles are all the same. So the interior angle at the top of the pentagon (underneath B) is also y. This means x and y lie on the same line. Use this to find y.

Answer:

$72° + y = 180°$ (angles on a straight line)

So

$y = 180° - 72°$

$y = 108°$

iii. New question:

Now see if you get the same answer for y using the sum of interior angles rule.

Answer:

Sum of interior angles is $(n - 2) \times 180°$.

Remember n = number of polygon sides. So:

Sum of interior angles = $(5 - 2) \times 180°$

Sum of interior angles = $540°$

$y = {}^{540°}\!/_{5}$

$y = 108°$

iv. Esmeralda's bearing when she turns to face B is 072° (since she turned the same amount as an exterior angle).

The next turn she makes is half of an interior angle, so ${}^{108°}\!/_{2} = 054°$.

This means in total her bearing is

$072° + 054° = 126°$.

6. Similar Triangles

a.

i. 1 reason: same bottom angles (based on diagram) OR same top angles because corresponding angles in parallel lines.

ii. Longer side of the smaller triangle = 3. Shorter side of smaller triangle = 2.

iii. Shorter side of bigger triangle = 2.5

iv. Longer side of bigger triangle = 3 + x

v. $\frac{\text{Longer side 1}}{\text{shorter side 1}} = \frac{\text{Longer side 2}}{\text{shorter side 2}}$

This means:

$\frac{3}{2} = \frac{3 + x}{2.5}$

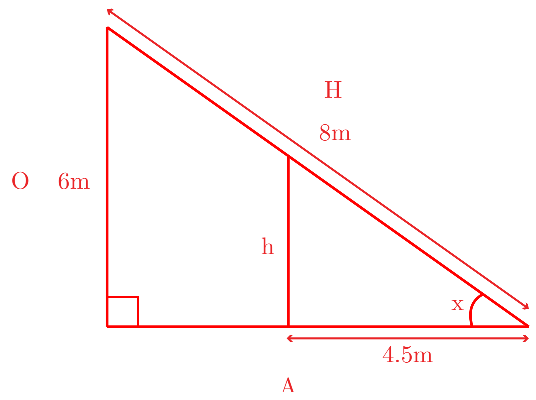
$3 + x = \frac{3}{2}(2.5)$

$x = \frac{3}{2}(2.5) - 3$

$x = 0.75$

b.

i.



ii. $\sin(x) = 6/8$

$\sin(x) = 0.75$

We want x by itself, so

$x = \sin^{-1}(0.75)$

$x = 48.6°$

iii. $h = \tan(48.6) \times 4.5$

$h = 5.1$

Section Two

1. Triangular Nacho Chips

a. The base angles in an isosceles triangle are equal, so we can calculate the third angle in the triangle by:

Top angle in triangle = $180° - 2(47°)$

Top angle in triangle = $86°$

So then

$x = 360° - 86°$ (angles at a point)

$x = 274°$

b. Using the same method as above:

Top angle in triangle = $180° - 2y$

$x = 360° - (180° - 2y)$

$x = 180° + 2y$

2. Horizontal Wires

a. $\angle CBD = 180° - 105°$ (angles on a line)

$\angle CBD = 75°$

$x = \angle CBD$ (corresponding angles on parallel lines)

$x = 75°$

Note you could have also used corresponding angles on parallel lines first, and then angles on a line. This should give you the same answer.

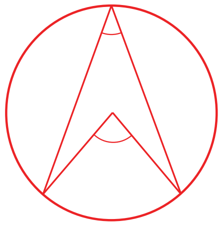
b. $\angle CDB = \angle CBD$ (since the triangle is symmetric, so it must be an isosceles triangle with equal base angles)

$\angle CDB = 75°$

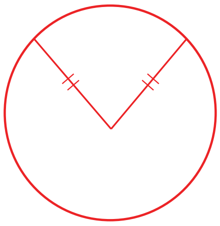
$\angle FIH = 75°$ (corresponding angles on parallel lines)

3. Stars in Circles

a.



Rule: Angle at centre = twice angle at circumference



Rule: Angles in triangle add to 180°. Isosceles base angles in triangle are equal.

b. $\angle DOC = 2(20)°$ (angles at centre is twice angle at circumference)

$\angle DOC = 40°$

$x = \angle DOC$ (vertically opposite angles)

$x = 40°$

c. Line OA = line OB because both are the radius of the same circle. So triangle OAB is isosceles. So using base angles in isosceles triangle and angles in a triangle,

$y = \frac{180° - 40°}{2}$

$y = 70°$

4. Polygons in Circles

a. Sum of interior angles = $(n - 2) \times 180°$

Sum of interior angles = $(6 - 2) \times 180°$

Sum of interior angles = $720°$

Because we have a regular polygon, all the interior angles are equal so

$a = {}^{720°}/_{6}$

$a = 120°$.

b. The triangle is isosceles (diagram shows two equal side lengths), so using base angles of isosceles and angles in a triangle we get:

$b = \frac{180° - 120°}{2}$

$b = 30°$

c. If the inscribed polygon has n sides, then

$a = \frac{(n - 2)180°}{n}$

$a = \frac{180°n - 360°}{n}$

$a = \frac{180° - 360°}{n}$

Then:

$b = \frac{180° - a}{2}$

$b = \frac{180° - \left(\frac{180° - 360°}{n}\right)}{2}$

$b = 90° - (90° - \frac{180°}{n})$

$b = \frac{180°}{n}$

5. Triangular Ramps

a. $\tan(\theta) = \frac{1.5}{3}$

$\tan(\theta) = 0.5$

$\theta = \tan^{-1}(0.5)$

$\theta = 26.57°$

Then to work out x, we calculate

$x = 7\sin(26.57)$

$x = 3.13m$

Section Three
Practice Exam

1. Question One

a.

i. $\tan(x) = \frac{147}{\left(\frac{230}{2}\right)}$

$\tan(x) = \frac{147}{115}$

$\tan(x) = 1.278$

So

$x = \tan^{-1}(1.278)$

$x = 51.96°$

Achieved for correct answer.

ii. Triangle ABC is isosceles, so

$y = 180° - 2(51.96)$ (angles in a triangle and base angles of isosceles)

$y = 76.1°$

Achieved for correct answer.

b. $\angle OBC = 45°$ because EDCB is a square, so it has angles of 90° which are cut equally in half when we draw a line to the opposite corner.

$s = \frac{115}{\cos(45°)}$

$s = 162.6\text{m}$

Merit for correct answer.

c.

i. Consider corner C. Note we could choose any corner because base is symmetric.

$\angle DCA = \frac{180°}{3}$ (angles in triangle and isosceles triangle).

$\angle DCA = 60°$

Then because line DA bisects the edge (cuts the base triangle in half), we have:

$\angle DAC = \frac{60°}{2}$

$\angle DAC = 30°$

So:

$\angle CDA = 180° - 60° - 30°$ (angles in a triangle)

$\angle CDA = 90°$.

Therefore all of the lines intersect the base triangle at 90°.

Excellence for correct answer.

ii. Pythagoras using the triangle CDA:

$b^2 = (\frac{b}{2})^2 + (2t)^2$

$b^2 = \frac{b^2}{4} + 4t^2$

$4t^2 = b^2 - \frac{b^2}{4}$

$4t^2 = \frac{3b^2}{4}$

$t^2 = \frac{3b^2}{16}$

$t = \frac{\sqrt{3}b}{4} \approx 0.433b.$

Or:

Trigonometry using triangle CDA:

$t = \cos(30°)\frac{b}{2}$

$t = 0.433b$

Note: you could have used any of sin/cos/tan since all three side lengths are known.

2. Question Two

a. $180° = x + 62° + 90°$ (angles on a line)

$x = 180 - 62° - 90°$

$x = 28°$

Achieved for correct answer.

b. $y = 90° + 28°$ (corresponding angles on parallel lines)

$y = 118°$

Achieved for correct answer.

c. The question says the diagram is symmetric, so the middle triangle must be isosceles.

Angle vertically opposite to $z = \frac{180° - 55°}{2}$ (angles in a triangle and angles in isosceles)

Angle vertically opposite to $z = 62.5°$

Therefore:

$z = 62.5°$ (vertically opposite angles)

Merit for correct answer.

d. Bottom left angle in quadrilateral = 180° – 70° (corresponding angles AND angles on a line).

Bottom left angle in quadrilateral = 110°

Bottom right angle = 180° – 42° (same reason).

Bottom right angle = 138°

Top left angle = 55° (corresponding angles AND vertically opposite angles).

So then:

a + 55° + 138° + 110° = 360° (interior angles of a quadrilateral)

a = 360° – 55° – 138° – 110°

a = 57°

Excellence for correct answer.

e. D is the midpoint of the semicircle which means triangle ADB is isosceles and triangle BDC is also isosceles. This means that ∠ABD = a and ∠DBC = b. Therefore c = a + b.

We also know a + b + c = 180° (angles in a triangle).

Replacing a + b with c in that equation (since we know c = a + b), we get

c + c = 180°

2c = 180°

c = 90°

Excellence for correct answer.

3. Question Three

a.

i. ∠DCE and ∠ACB vertically opposite and are therefore equal. ∠CDE and ∠ABC are alternate in parallel lines and are also equal. Both triangles also have right angles and so all their angles are the same, and therefore they are similar.

Achieved for one justification given.

Merit for two justifications given.

ii. ∠CDE = 70° (alternating angles on parallel lines)

Achieved for correct answer.

b. $w = \frac{180}{\tan(70°)}$

w = 65.51m

Achieved for correct answer.

c. $l = \frac{41}{\sin(70°)}$

$l = 43.63$ m

Merit for correct answer.

d. Firstly, notice AB = CD because of parallel lines.

To find CD:

$40^2 = 30^2 + (CD)^2$ (using Pythagoras)

$CD = \sqrt{40^2 - 30^2}$

CD = 26.458m.

Then let x = vertical height of bottom (bigger) triangle

$230 = x^2 + 26.458^2$

$x = \sqrt{230^2 - 26.458^2}$

x = 228.473m.

Lastly, looking at the triangle on the left, we subtract CD from 325 to get the bottom length of the triangle, then use Pythagoras to get:

$h = \sqrt{(325 - 26.758)^2 - 228.473^2}$

h = 375.9m

Excellence for correct answer.