MCAT
ANSWERS

The Language of Maths

In Terms of
STOP AND CHECK (PAGE 6)

- We need to be able to speak and write maths to describe the world around us and communicate mathematical ideas in a universal language.
- An expression is a collection of mathematical symbols that represent something and obey mathematical rules such as BEDMAS. An equation is two expressions connected with an equals (=) sign.
- 'In terms of' means the answer should have one variable isolated on one side of the equation. If you read an equation left-to-right, the variable on the left is being put in terms of the right side of the equation.

Algebra Basics

Basic Operations
STOP AND CHECK (PAGE 12)

- $b = \frac{a}{5} - 10$:

$$b + 10 = \frac{a}{5}$$
$$5(b + 10) = a$$
$$a = 5(b + 10)$$

- $c = 2a + 3b - 7$:

$$c - 2a = 3b - 7$$

$$\frac{c - 2a + 7}{3} = b$$

Linear Inequalities

STOP AND CHECK (PAGE 14)

- $7 > 3 - 5y$:

$$7 - 3 > -5y$$

$$4 > -5y$$

$$\frac{4}{-5} < y$$

- An important rule to bear in mind when solving inequalities is to reverse the inequality symbol if you are multiplying or dividing by a negative number.

Factorising and Expanding

STOP AND CHECK (PAGE 16)

- To simplify and rearrange:
    1. Expand brackets (if they're in the equation)
    2. Gather all like terms on one side
    3. Isolate *x* (or the unknown variable you are finding)
    4. Simplify
- $(3g - 9) - 4(g + 2) > 0$

$$3g - 9 - 4g - 8 > 0$$

$$-17 - g > 0$$

$$-17 > g$$

Factorising and Expanding

STOP AND CHECK (PAGE 16)

- Steps to take to simplify and rearrange:
    1. Expand brackets (if they're in the equation).
    2. Gather all like terms on one side.
    3. Isolate *x* (or the unknown variable you are finding).
    4. 4. Simplify.
- $(3g - 9) - 4(g + 2) > 0$:

$$3g - 9 - 4g - 8 > 0$$
$$-17 - g > 0$$
$$-17 > g$$

How to Form and Solve Linear Equations and Inequalities

STOP AND CHECK (PAGE 18)

- For this question, we can make the variable P represent the number of jars of peanut butter that Tasmin buys. Then we know that 4.90 × P is how much Tasmin spent on peanut butter since a jar of peanut butter is $4.90. So, using that we can write the following equation:

$$8.35 + 4.9P = 42.65$$
$$4.9P = 34.3$$
$$P = \frac{34.3}{4.9}$$
$$P = 7$$

So, Tasmin bought 7 jars of peanut butter.

- This question is a bit of a tricky one because we're not going to immediately have an equation for how much milk Skip is drinking. First off, we have to write an equation about how long the two are drinking, which will look like this:

  $$t(14 \times 2 + 14 + 4) = 138$$

  Where t represents how long it takes Skip to pass out. If you look closely at this equation, the first part inside the brackets represents how much Bo is drinking since he'll be drinking twice as long as Skip, so his 14L/hr (litres per hour) is doubled. The second part in the brackets represents how much Skip drinks since he drinks 4L/hr more than Bo, which is 14 + 4 = 18L/hr. We multiply both these numbers by time because if you're drinking 46 litres of milk every hour, when you multiply that by how many hours you're drinking you get the number of litres you'll drink. Now we can solve the equation for time:

  $$t(46) = 138$$
  $$t = \frac{138}{46}$$
  $$t = 3$$

  Now that we have time, we can multiply Skip's litres per hour by the time, which looks like:

  $$18 \times 3 = 54L$$

  So, Skip drinks 54L of milk.
- To start off, let E represent the number of hours Ethan mucks around and J be the number of hours Jake mucks around. Ethan is saying that he mucks around more than Jake, which in math terms is represented by:

  $$E > J$$

  The question asks to check the accuracy of the statement. We know Jake mucks around 3 hours a day, 4 days a week which adds to 12 hours a week. So J = 12.

Ethan says he mucks around for two-thirds of those 12 hours, as well as another 2 hours on the weekend:

$$12 \times \frac{2}{3} = 8$$

$$8 + 2 = 10$$

$$E \geq 10.$$

So, Ethan isn't always right since some weeks he only mucks around for 10 hours, but there are some weeks where he does slack off more than Jake.

Algebraic Fractions

Multiplying and Dividing

STOP AND CHECK (PAGE 22)

- $\frac{16x}{6}$

$$\frac{2(8x)}{2(3)}$$

$$\frac{8x}{3}$$

- $\frac{4-x}{\frac{5}{6}x}$

$$\frac{4-x}{\frac{5}{6}x} = \frac{4-x}{x} \times \frac{1}{\frac{5}{6}}$$

$$\frac{4-x}{x} \times \frac{6}{5}$$

$$\frac{6(4-x)}{5x}$$

$$\frac{24-6x}{5x}$$

So what we've done here is take apart the fraction into pieces that are a bit easier to manage, then expanded everything out into one neat fraction.

How to Add, Subtract and Multiply Fractions

STOP AND CHECK (PAGE 25)

- You draw our friend, the upside-down picnic table.
- Just multiply across the numerators and denominators. We can write this algebraically as well, given two fractions $\frac{a}{b}$and $\frac{c}{d}$then:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Rearranging Algebraic Fractions

STOP AND CHECK (PAGE 28)

- Make x the subject of $\frac{2x+3}{3} - \frac{4x-2}{5} = 0$

  For this, we're gonna use our upside-down picnic table.

$$\frac{5(2x+3)-3(4x-2)}{3\times 5} = 0$$

$$\frac{10x+15-12x+6}{15} = 0$$

$$\frac{-2x+21}{15} = 0$$

$$-2x + 21 = 0$$

$$-2x = -21$$

$$x = 10.5$$

- Solve for x $\frac{1}{x} + \frac{1}{4} = \frac{1}{3}$

  For this, we're gonna multiply through by a common multiple. In this case, $12x$.

$$\frac{12x}{x} + \frac{12x}{4} = \frac{12x}{3}$$

$$12 + 3x = 4x$$

$$12 = x$$

Algebraic Fractions

QUICK QUESTIONS (PAGE 29)

- $\frac{2}{x} + \frac{2+x}{3} = \frac{x^2+2x+6}{3x}$

$$\frac{2}{x} + \frac{2+x}{3}$$

$$\frac{6}{3x} + \frac{x^2+2x}{3x}$$

$$\frac{x^2+2x+6}{3x}$$

- Write as a single fraction:
  Multiply the numerator by the denominator from the other fraction. This is to find a common multiple which will then become the denominator:

$$\frac{2x-4}{(x+1)(x-2)} + \frac{3x+3}{(x+1)(x-2)}$$

  Combine the fractions that now have the same denominator:

$$\frac{5x-1}{(x+1)(x-2)}$$

  Expand the denominator to remove the final bracket:

$$\frac{5x-1}{x^2-x-2}$$

- Simplify: $\frac{x^2-x-2}{5x^2-10x}$
  Factorise the numerator and denominator:

$$\frac{(x+1)(x-2)}{5x(x-2)}$$

  Cancel our like terms, in this case, *x* - 2:

$$\frac{x+1}{5x}$$

Quadratics

Defining and Drawing Quadratics

STOP AND CHECK (PAGE 30)

- A parabolic curve/parabola.

- They have an x^2 variable which means there are two possible solutions for x.

How to Factorise and Expand Quadratics

STOP AND CHECK (PAGE 35)

- Factorise the following:
  - $x^2 - 3x + 2$

$$(x - 3)(x - 1)$$

  - $x^2 + 2x - 35$

$$(x + 7)(x - 5)$$

  - $x^2 - 9$

$$(x + 3)(x - 3)$$

  - $x^2 - 64$

$$(x + 8)(x - 8)$$

  - $3x^2 + 14x + 8$

$$3 \times 8 = 24$$

$$3x^2 + 12x + 2x + 8$$

$$(3x^2 + 12x) + (2x + 8)$$

$$3x(x + 4) + 2(x + 4)$$

$$(3x + 2)(x + 4)$$

How to Solve Quadratic Equations

STOP AND CHECK (PAGE 40)

- Solve the following:
  - $x^2 - 8x + 12 = 4x - 8$

$$x^2 - 12x + 20 = 0$$

$$(x - 2)(x - 10)$$

$$x = 2, 10$$

  - $6x - x^2 + 3 = -4$

$$-x^2 + 6x + 7$$

$$(-x - 1)(x - 7) = 0$$
$$x = -1, 7$$

Substituting Numbers into Quadratic Equations

STOP AND CHECK (PAGE 44)

- $(x - 3)(x + 3) > (x - 2)(x + 3)$

$$x^2 + 3x - 3x - 9 > x^2 + 3x - 2x - 6$$
$$x^2 - 9 > x^2 + x - 6$$
$$-9 > x - 6$$
$$-3 > x$$
$$x < -3$$

Quadratics

QUICK QUESTIONS (PAGE 44)

- Height above the ground when $x = 2$:

$$y = 2 \times 2(2 - 1.5)$$
$$y = 4(0.5)$$
$$y = 2$$

- What values of x is y negative?

$$y = x^2 + 5x + 6$$
$$y = (x + 2)(x + 3)$$
$$x = -2, -3$$

Exponents

The Laws of Exponents/Indicies

STOP AND CHECK (PAGE 46)

- $x \times x \times x \times x \times x$
- Four rules:
    - Any variable to the power of 0 will equal 1.
    - When multiplying powers with the same base, add the exponents.
    - When dividing, minus the powers.
    - When there's a power inside a power, multiply them to expand the brackets.

Using Indicies in Equations

STOP AND CHECK (PAGE 48)

- The golden rule is to make everything the same base.
- $x = 12$

How to Solve Equations with Exponents

STOP AND CHECK (PAGE 49)

- Ask yourself, what number when multiplied by itself 3 times makes 27? That's 3.

$$3 \times 3 \times 3 = 27$$

$$x = 3$$

- $n = \sqrt[4]{256}$

$$4 \times 4 \times 4 \times 4 = 256$$

Exponents

QUICK QUESTIONS (PAGE 50)

- As we want to know the days and already have the time goal, we'll chuck the 160 in for T in our equation and solve:

$$T = 10 \times 2^{n-1}$$
$$160 = 10 \times 2^{n-1}$$
$$16 = 2^{n-1}$$
$$2^4 = 2^{n-1}$$
$$4 = n - 1$$
$$N = 5$$

It will take 5 days to reach her goal of 160 minutes a day.

Simultaneous Equations

What Simultaneous Equations Are

STOP AND CHECK (PAGE 51)

- The point of intersection is the point of interest when you're dealing with two equations at the same time.

Rearranging Simultaneous Equations

STOP AND CHECK (PAGE 54)

- For this one, it'll be useful to use our elimination method; now I like to line the equations one on top of the other to make it a bit clearer, but you can put them on the same line if you want.

$$3a - b = 5$$
$$2x + 2b = 6$$

For one of our variables to cancel, we can multiply through the top line by 2 to get a 2b. Then, we can just add the two equations for the b's to cancel.

$$6a - 2b = 10$$
$$+(2a + 2b = 6)$$
$$6a - 2b + (2a + 2b) = 10 + (6)$$

Here we've just put everything that came from the second equation in brackets so:

$$8a = 16$$
$$a = 2$$

Now with our a, we can substitute it back into one of our original equations to find b:

$$3(2) - b = 5$$
$$6 - b = 5$$
$$b = 1$$

- For this equation, we can use our substitution. This one is a little bit more difficult to see, but if we double the second equation, you'll see that:

$$\frac{2x}{3} + 4y = 12$$
$$4y - 12x = 12$$

So since both equations equal 12, we can just set the left hand sides equal to each other right now, but that won't always work – here it will work because you'll have 4y on both sides so they'll cancel and you'll only have x in the equation, but it won't always work, so to be safe we're going to do a bit more rearranging:

$$\frac{2x}{3} + 4y = 12$$
$$4y = 12x + 12$$

Now, we can substitute that $4y = 12 + 12x$ in our other equation and solve:

$$\frac{2x}{3} + (12 + 12x) = 12$$

Notice how everything in the brackets is what we switched for our 4y. Now, we can solve:

$$\frac{2x}{3} + 12x = 0$$

From here, we can see that x = 0. We can now substitute this into our original equations to find y:

$$2y - 6(0) = 6$$
$$2y = 6$$
$$y = 3$$

Solving Simultaneous Equations that Include a Parabolas

STOP AND CHECK (PAGE 59)

- For this equation we can equate the equations easily, since y = 2x + 1 and y = x^2 - 2x + 5 to get:

$$2x + 1 = x^2 - 2x + 5$$

Then we can do some rearranging and solve:

$$x^2 - 4x + 4 = 0$$
$$(x - 2)(x - 2) = 0$$

We can see that x = 2. We can now substitute this into our original equation to get:

$$y = 2(2) + 1$$
$$y = 5$$
$$x = 2$$

- For this question we're going to do the same thing as the previous question, replacing our y's with the other side of their equations, like this:

$$3x = x^2 + 6x - 10$$
$$x^2 + 3x - 10 = 0$$
$$(x - 2)(x + 5) = 0$$
$$x = 2, -5$$

Simultaneous Equations

QUICK QUESTIONS (PAGE 59)

- This question is one of our old simultaneous questions, but for this one we're not going to be able to write it all in terms of one variable, so we're going to have to create two variables: C for the number of children that buy lemonade, and A for the number of adults that buy lemonade. Then we can see from the question that in total 272 people buy lemonade, so we can write:

  $$A + C = 272$$

  We also know that \$578 was collected, and the cost of lemonade is \$1.50 for children and \$4.00 for adults, so we know that:

  $$1.5C + 4A = 578$$

  Which we can then substitute into our second equation in place of A then solve, like so:

  $$1.5C + 4(272 - C) = 578$$
  $$1.5C + 1088 - 4C = 578$$
  $$-2.5C = -510$$
  $$C = 204$$

  Now, we can substitute this into our first equation:

  $$A + 204 = 272$$
  $$A = 68$$

  So, 204 children and 68 adults bought lemonade from the lemonade stand. It's important to add that last line to relate your answer back to the context of the question and show you know what that answer means, that’s how you get Excellences.