Workbook Answers
Level 1 Maths
MCAT

Section One
Part One

1. BEDMAS

So for each of these we just have to be really careful that we apply all of the operations in the correct order as per BEDMAS. We will go through most of the steps for the first few and then leave the rest up to you!

i. $2 \times (2) + (4)^2 = 20$

ii. $\frac{2}{4} = 0.5$

iii. $3 \times (\frac{4}{2}) = 6$

iv. $0.5 \times 5 - 2 = -5$

v. $\frac{18 + 9 - 23}{4} = 1$

vi. $16 + 4 - 10 = 10$

vii. $2(4 - 2 - 1) = 2$

viii. 2

2. Fractions

a. We need to remember the rules for adding, subtracting, multiplying and dividing fractions.

i. $\frac{2}{3}$

ii. $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

iii. $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

iv. $2 \times \frac{2}{7} = \frac{2}{1} \times \frac{2}{7} = \frac{4}{7}$

v. $\frac{1}{5} + 2\frac{1}{3} = \frac{3}{15} + 2\frac{5}{15} = 2\frac{8}{15}$

vi. $\frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = \frac{2}{15}$

vii. $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

viii. $\left(\frac{1}{3} + \frac{1}{2}\right)^2 = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$

ix. $\frac{2}{(^1/_2)^2} = \frac{2}{(^1/_4)} = 2 \times 4 = 8$

x. $6\left(\frac{1}{4} \div \frac{1}{2}\right) = 6\left(\frac{1}{4} \times 2\right) = 3$

b. Writing the equations for this might be confusing so we will explain it in words too. To add or subtract a fraction, they need to have the same denominator, so:

$$\left(\frac{a}{b} + \frac{c}{b}\right) = \left(\frac{a + c}{b}\right)$$

And the same is true for subtracting. To multiply fractions, we don't need the same denominator. Instead we just multiply across the bottom and the top like $\frac{a}{b} \times \frac{c}{b} = \frac{ac}{bd}$. To divide fractions, we get $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$. In words, we flip the fraction that is dividing, and multiply normally.

3. Like Terms, Simplifying and Factorising

a. This section tests whether you can identify like terms and common factors in expressions.

i. $x + 2x$
$3x$

ii. $3ab + 2a - ab$
$2ab + 2a$
$2a(b + 1)$

iii. $2x + 3x - x + 4$
$4x + 4$
$4(x + 1)$

iv. $2t^2 + 4$
$2(t^2 + 2)$

v. $10x - 4x + 3 + 2y - 2x + 4$
$4x + 2y + 7$

vi. $n^3 + 2n^2 - 3n^2 + 5n$
$n(n^2 - n + 5)$

vii. $3y + 2(y + 3) - y(y + 2)$
$3y + 2y + 6 - y^2 - 2y$
$-y^2 + 3y + 6$ or
$y^2 - 3y - 6$

viii. $3x + 4 - 2x + 1 - (x - 1)$
$x + 5 - x + 1$
6

ix. $2ab + 2ab^2$
$2ab(1 + b)$

x. $3x^2 + 6x$
$3x(x + 2)$

4. Perimeters of Shapes

The perimeter of any shape is just the total length on the outside of the shape. For all of these answers, we let P be the perimeter of the given shape.

a. i. $P = 3 + 4 + 5 = 12$

ii. $P = 3 + 3 + 6 = 12$

iii. $P = 3 + 3 + 3 = 9$

iv. $P = x + 4 + 3 = x + 7$

b. i. $P = 3 + 3 + 3 + 3 = 12$

ii. $P = 5 + 5 + 5 + 5 = 20$

iii. $P = 4x$. When $x = 2$, $P = 8$

iv. $P = 8x$. When $x = 3$, $P = 24$

c. i. $P = 8$

ii. $P = 14$

iii. $P = 6x$. When $x = 1$, $P = 6$

iv. $P = 6x$. When $x = 3$, $P = 18$

d. A pentagon has 5 sides, so the perimeter is the sum of these sides. If all the sides are the same, it could be given by the expression $5x$. A hexagon has 6 sides, so the perimeter is the sum of these sides. If the sides are the same, it could be given by the expression $6x$.

e. The perimeter of a shape with n number of sides and a side length of x is given by the expression nx. Otherwise, it is the sum of the lengths of n number of sides.

f. The perimeter of a circle is called the circumference, which can be found using the formula $C = 2\pi r$.

5. Areas of Shapes

a. The area of a shape is the amount of space enclosed by the shape in two dimensions. There are many ways to define the area of a shape but they all say roughly the same thing.

b. i. $A = \frac{1}{2}\,\text{base} \times \text{height}$

ii. $A = x^2$ where x is the length of the side

iii. $A = \text{length} \times \text{width}$

iv. $A = \pi r^2$ where r is the radius of the circle

c. For some of these answers, the shapes have been split into parts.

i. $A = 3^2 = 9$

ii. $A = \frac{1}{2} \times 3 \times 5 = 7.5 \text{ or } \frac{15}{2}$

iii. $A = 6 \times 9 = 54$

iv. $A = \pi(3)^2 = 9\pi$

v. $A = (4 \times 7) + (0.5 \times 4 \times 3) = 34$

vi. $A = (3 \times 6) + (4 \times 2) = 26$

vii. This is a bit of an 'extra for experts' question. We need to use Pythagoras' to get the height of the triangle, which is:

$\sqrt{(25 - 6.25)}$.

So the area of the equilateral triangle is:

$2.5\times\sqrt{(19.75)}$, so
$A = 20 + 2.5\times\sqrt{(19.75)} = 30.825$

(with a calculator which isn't allowed!)

viii. $A = x^2$

Part Two

1. Linear Equations and Inequations

a.

i. $2x = 16$
$x = 8$

ii. $x - 5 = 9$
$x = 14$

iii. $3x - 5 = 2x + 1$
$x = 6$

iv. $3t - 2(t + 1) = 4$
$3t - 2t - 2) = 4$
$t = 6$

v. $2x + 1 - 4x + 3\,(2x + 4) = 3$
$2x + 1 - 4x + 6x + 12 = 3$
$4x = -10$
$x = \frac{-10}{4}$

vi. $\frac{2x + 4}{2} = 4$
$2x + 4 = 8$
$2x = 4$
$x = 2$

vii. $3w = 7$
$w = \frac{7}{3}$

viii. $\frac{16x^2}{x} - 12x = 2$
$16x - 12x = 2$
$4x = 2$
$x = 0.5$

ix. $20x + 30\,(x - 2) = 540$
$20x + 30x - 60 = 540$
$50x = 600$
$5x = 60$
$x = 12$

x. $\frac{1}{2}x + \frac{1}{3}x = 10$
$\frac{5}{6}x = 10$
$5x = 60$
$x = 12$

b. An inequation is a statement about the size of two things when compared to each other. They are useful when trying to understand how different expressions relate to each other. They use the symbols > 'greater than', < 'less than', $\geq$ 'greater than or equal to' and $\leq$ 'less than or equal to'.

c. This may be confusing, just ask yourself, is the thing on the left 'greater than', or 'less than' the number on the right. It really helps if you actually make substitutions, which is why we gave you a bit of space to have a play around. Some of these are almost true. Especially the last couple, there may only be one value that makes the inequation false, which means the entire thing is false!

i. Not true. Could change sign to $<$

ii. This is true.

iii. This is true.

iv. This is not true. We find that it's actually when $x > \frac{1}{2}$

v. This is true.

vi. This is true.

vii. This is true.

viii. This is not true. The only value that x can't be is 0, so we could say that $x \neq 0$ or change sign to $\geq$

ix. This is not true because x could equal 0. Could change sign to >

x. This is true.

d.

i. $x > 2$

ii. $3x > 15$
$x > 5$

iii. $2y + 2 < 24$
$y < 11$

iv. $2x \geq 12$
$x \geq 6$

v. $t < 3$

vi. $x > 23$

vii. $x < 2$

viii. Divide both sides by $3t$, so $t + 1 < 1$, which means that $t < 0$.

ix. Rearranging:

$$\frac{x + 6}{x - 3} > -2$$
$$x + 6 > -2(x - 3)$$
$$x + 6 > -2x + 6$$
$$\frac{3x}{3} > \frac{0}{3}$$
$$x > 0, \text{ but } x \neq 3$$

e. This part is all about translating words into maths, this is worth practising a lot. Think hard about the language we use, you could even write up a translation dictionary for certain words like 'the result of' and 'product'.

i. Let the number be n. The answer is $3n > 15$, so $n > 5$.

ii. We get the inequality $3n - 5 > 2n + 10$ and rearrange for n, which is $n > 15$.

iii. Two times the result of t plus three is $2(t + 3)$and the inequality is $2(t + 3) > 86$. After rearranging we get $t + 3 > 43$, so $t > 40$.

iv. $6x < 4(x+3)$. Rearrange to get $x < 6$.

v. $-3x > -9$ so $x < 3$.

vi. Let x be the number of hours. The inequality is $5x + 15 < 8x$ and we rearrange to get $15 < 3x$ so $x > 5$. We would have to rent for more than 5 hours for company 1 to be cheaper.

2. Rearranging, Substitution and Writing Maths

a. Rearranging equations is covered in the first section of our Walkthrough Guide, although this also includes some skills from later sections.

i. $y = 0.5x + 2$

ii. $t = y + 1$

iii. $y = 2x - 3$

iv. $y = \frac{4}{3} - \frac{7}{3}x$

v. $x = 2 + 0.5y$

vi. $A = 2 - x - y$

vii. $x = \sqrt{y}$

viii. $y\,(3x + 3x^2) = 2x + 5$ so
$y = \frac{2x + 5}{3x + 3x^2}$

b. For these we just have to be careful when we make our substitutions. In the later questions, we may have to make some smart rearrangements after doing our substitution.

i. $y = 3(2) - 4 = 2$

ii. $A = 3(3 - 1) = 6$

iii. $(3)^2 + 4(3) = 21$

iv. $d = 20$

v. $A = (2(2) + 1)^2 = 5^2 = 25$

vi. $6 - (x + 1) = 2x + 5$
$6 = 3x + 6$
$x = 0$

vii. $y = (4)(1) = 4$

viii. $P = 3^4 = 81$

ix. $\sqrt{9} + 2(9) - 5 = n$ when $m = 9$
$3 + 18 - 5 = n$
$16 = n$

x. $y = -16$

xi. $\frac{6 + y}{x + 4} = 2$
$\frac{6 + 2}{x + 4} = 2$
$x = 0$

xii. $64 = r^3$
$r = 4$

c. We have switched up the variables we use to answer these questions because the letter we use doesn't matter, as it's just a symbol to represent a number.

i. $4n$

ii. $6n = 12$

iii. $\frac{1}{2}\,(n + 2)$

iv. $2(x + 1) = 20$

v. $10x$

vi. $a + b$

vii. $3z$

viii. $a + (a + 1)$, which can also be written as $2a + 1$

ix. x^2

x. $x^2 - 36 = 0$

d. i. $5n = 4n + 1$
$n = 1$

ii. $15 + 5n = 75$
$n = 12$

iii. $2(n + 1) = 20$
$2n + 2 = 20$
$2n = 18$
$n = 9$

iv. $\frac{n}{4} = 9$
$n = 36$

v. $\frac{3n + 1}{2} = 5$
$3n + 1 = 10$
$3n = 9$
$n = 3$

e. i. A counting number cannot be less than 0. These are sometimes called the natural numbers. On the other hand, an integer can be less than 0.

ii. An even number is divisible by 2 (that is, dividing by 2 gives a whole number). We can make sure this happens by writing $2n$. Although, we should probably say that n can't be zero because 0 isn't even or odd (it's just 0).

iii. The number can't be zero and must be greater than 0, so n has to be greater than, or equal to 1. So the smallest value of n is 1, which makes sense because 2 is the smallest, positive whole number!

iv. Any whole odd number is just one more than any whole even number. So, we can write $2n + 1$ or $2n - 1$.

v. For both $2n + 1$ and $2n$ - 1, the smallest value of n is 1 because $n = 0$ is excluded and $n = 1$ still gives a positive, whole number for both expressions.

vi. $2n + 2n - 1$

$= 4n - 1$

Because 4 is divisible by 2 and we are subtracting 1, there is no value of n that gives an even number. Therefore, adding an even number to an odd number always gives an odd number.

vii. For this, our first odd number is $2n - 1$ and our other odd number is $2n + 1$. Adding these together gives $4n$, which we know must be even because it is divisible by two.

3. Quadratic Equations and Expressions

a. There are actually a few general forms of quadratic equations, but the important one for the MCAT is the form $y = ax^2 + bx + c$. This can also appear in the factorised form as $y = k(x + p)(x+ q)$. They are different from linear equations in many ways. For example, they have a variable squared in their equation, they don't form a straight line, and they often have two solutions, amongst other things.

b. i. Quadratic

ii. Not a quadratic

iii. Quadratic

iv. Quadratic

v. Quadratic

vi. Not a quadratic

vii. Quadratic

viii. Quadratic

c. i. $a = 1$, $b = 5$ and $c = 6$.

ii. $a = 2$, $b = -8$ and $c = 16$

iii. $a = 1$, $b = 0$ and $c = -16$

iv. $a = 2$, $b = -3$ and $c = 0$

v. $a = 3$, b and c are 0.

vi. $a = 15$, $b = -12$ and $c = 6$

vii. $a = 2$, $b = 2$ and $c = 6$

viii. $a = -3$, $b = 3$ and $c = -15$

ix. $a = 3$, $b = 12$ and $c = 8$

d. The solutions to a quadratic equation when solving for x will show you the x-intercepts of the parabola. This is where the parabola crosses the x-axis (the horizontal one).

e. Factorise the following quadratic equations or expressions. If it's an equation, make sure to write in the form $ax^2 + bx + c = 0$ before factorising.

i. $(x + 3)(x + 4)$

ii. $(x + 10)(x - 3)$

iii. $3x(x + 1) = 0$

iv. $(x + 4)(x - 4)$

v. $(x + 2)(x + 2)$ or $(x + 2)^2$

vi. $2(x - 4)(x + 2)$

vii. $3(x - 2)(x + 4)$

viii. $(x - 6)(x + 6)$

ix. $x(7x + 3)$

x. $2(x + 1)(x - 1)$

xi. $2x^2 + 14x + x + 7$
$2x(x + 7) + (x + 7)$
$(2x + 1) + (x + 7)$

xii. $3x^2 + 12x - 2x - 8$
$3x(x + 4) - 2(x + 4)$
$(3x - 2) + (x + 4)$

f. i. $x = -2$
$x = -3$

ii. $x = 1$
$x = -2$

iii. $x = -3$
$x = 10$

iv. $x = 4$
$x = -4$

v. $2(x^2 + x - 2) = 0$
$(x + 2)\ (x - 1) = 0$
$x = -2, 1$

vi. $x^2 + 3 = 2x^2 + 2x + 4$
$(x + 1)\ (x + 1) = 0$
$x = -1$

vii. $(x + 3)\ (x + 4) = 0$
$x = -3, -4$

viii. $3x(x + 1) = 0$
$x = 0, x = -1$

ix. $(x - 3)\ (x + 1) < 0$
$-1 < x < 3$

x. $-2 < x < 0$

xi. $(x + 6)\ (x - 6) < 0$
$-6 < x < 6$

xii. $x(x - 4) < 0$
$0 < x < 4$

xiii. $5x(x - 5) + 3(x - 5) = 0$
$(5x + 3)\ (x - 5) = 0$
$x = \frac{-3}{5}, 5$

xiv. $-2x^2 + 2 > 0$
$x^2 < 1$
$-1 < x < 1$

xv. $5x^2 - 22x - 15 = 0$
$x = \frac{-3}{5}, 5$

g. When we expand $(x + a)(x + b)$ by multiplication, we get:
$x^2 + ax + bx + ab$
We can then factorise the ax and bx as:
$x(a + b)$
Put together, this creates:
$x^2 + x(a + b) + ab$
We can use this expression to factorise quadratics by comparing the coefficients of a quadratic with a and b.

4. Algebraic Fractions (Rational Expressions)

a. A rational expression is one polynomial (usually a quadratic which has the highest power of 2) in the numerator and/or denominator of a fraction. To simplify them, we need to be good with fractions and be able to factorise, expand, and simplify quadratics.

b. i. 2

ii. $x + 3$

iii. $\frac{x}{2}$

iv. $\frac{3(x-2)}{(x+3)\ (x-2)} = \frac{3}{(x+3)}$

v. 2

vi. $x - 4$

vii. $\frac{4(x+2)\ (x+3)}{2(x+2)\ (x+3)} = 2$

viii. $\frac{3x-6}{x-3}$

ix. 1

x. $\frac{(x-7)\ (x-1)}{(x-7)\ (x-3)}$

$\frac{(x+1)}{(x-3)}$

xi. 1

xii. $\frac{(2x+1)\ (x-4)}{(2x+1)\ (x+7)}$

$\frac{(x-4)}{(x+7)}$

c. i. $\frac{4}{x}$

ii. $\frac{8}{(x+2)}$

iii. $2x$

iv. $\frac{2x^2+12}{3x}$

v. $\frac{x(x-1)}{4}$

vi. $\frac{2}{x-2}$

d. i. $x = 6$

ii. $x = 8$

iii. $x = 8$

iv. $x^2 + 4x + 4 = 0$

$(x+2)\ (x+2) = 0$

$x = -2$

v. $\frac{3}{x} = \frac{3}{2}$

$x = 2$

vi. $x - 2 = 4$

$x = -1$

$x = 6$

vii. $(x+5)(x+2) = 2x(x+5)$

$x + 2 = 2x$

$x = 2$

$x = -5$

5. Simple Exponent Problems

a. An exponential equation or expression contains a number (the base) to a power (the exponent). The power can be a variable or a number. These types of equations can model growth or decay, like the growth of bacteria, compounding interest, or the depreciation (or loss) of a car's value.

b. i. Yes. Base = 3, Power = 2.

ii. No.

iii. Yes. Base = 3, Power = x.

iv. Yes. Base = 3, Power = x.

v. No.

vi. No.

vii. Yes. Base = 5, Power = x.

viii. No.

ix. No.

x. Yes. Base = 3, Power = t.

c. i. 2^5 ii. 5^4 iii. $3^{(2+x)}$

iv. $2^{(2+x)}$ v. 2^{2x} vi. 6^7

vii. $3^{(x+1)}$ viii. 3^{2x}

d. i. The rule is $x^a \times x^b = x^{a+b}$, as long as x are the same.

ii. The rule is $(x^a)^b = x^{ab}$

iii. $x^a \div x^b = x^{a\text{-}b}$. Note that the order of subtraction is really important.

e. i. $x = 2$

ii. $2^2 \times 2^x = 2^6$
$2 + x = 6$
$x = 4$

iii. $3^x = 3^4$
$x = 4$

iv. $6^x = 36$
$6^x = 6^2$
$x = 2$

v. $2^{(x+1)} = 2^5$
$x + 1 = 5$
$x = 4$

vi. $3^{6x} = 3^4$
$6x = 4$
$x = \frac{2}{3}$

vii. $4^x = 16 = 4^2$
$x = 2$

viii. $4^{2x} = 4^4$
$x = 2$

ix. $x = 2$

6. Simultaneous Equations

a. A simultaneous equation is two linear equations where the variables are the same. They allow us to find the values of the variables that satisfy both of the equations.

b. i. Subtracting the second equation from the first gives $2x = 6$ so $x = 3$. Substituting this into either equation gives $y = 1$.

ii. Subtracting the second equation from the first gives $b = 2$ and substituting this into either gives $a = \frac{3}{2}$.

iii. We multiply the second equation by 2. Then we subtract the first equation from the second to give $7y = -28$ so $y = -4$. Any equation gives x=2.

iv. Multiply the second equation by 2. Subtract the second from the first to give $y = 8$. Substituting this into the second gives $x = 1$.

v. Add the second equation to the first to get $7y = 7$ so $y = 1$. Substituting this into any other equation gives x = -1.

vi. Subtracting the second equation from the first we get $3y = 12$ so $y = 4$. Substituting this into any other equation gives $x = 5$.

c. i. Rearrange equation one to get $x = 14 - 2y$ and substitute this into the second to get $3y + 2(14 - 2y) = 22$ so $y = 6$. Therefore $x = 2$

ii. Rearranging the first equation we get $2y - 2 = x$, substituting this into the second we get $-4y + 3(2y - 2) = -4$, rearranging for y we get $y = 1$. Substituting into the first equation gives x = 0.

iii. Rearranging the first equation, we get 3y = 3x – 30. Substituting this into equation two to gives 2x + 3x – 30 = 30, which means x = 12 and y = 2.

iv. We can set these equations equal to one another so $x + 2 = -\frac{1}{3}x + 6$. We rearrange this (we have to subtract fractions) to get $x = 3$. So $y = 5$.

v. Rearrange the second equation to get $y = 7 - x$ and substitute this into the first to get $x = 4$ so $y = 3$

d. i. Let a be the number of cookies sold and b be the number of muffins sold. So the amount made is $a + 3b = 40$ and the total number is $a + b = 20$. Rearranging the second equation we get $a = 20 - b$.

Substituting this into the first equation give $b = 10$ so $a = 10$. Overall you sold 10 cookies and 10 muffins.

ii. Let h be the amount of money Huia has and k be the amount that Kira has. Then $h + k = 30$. We can write the first bit of information as $k + 10 = 2(h - 10)$. We will expand out the second equation and rearrange for k in terms of h which is $k = 2h - 30$. Substituting this into the first equation gives $3h = 60$ so $h = 20$ and $k = 10$.

iii. The equations are $b + 3 = a - 4$ and $a = 2b$. Substituting this straight into the first equation we get $b + 3 = 2b - 4$ so $b = 7$ and so $a = 14$.

iv. Let the number of books in the first bag be a and the number of books in the second bag be b. Then $a = \frac{1}{2}b$ and $b - 4 = a$. Substituting this into the first equation we get $b - 4 = \frac{1}{2}b$, so $\frac{1}{2}b = 4$ and $b = 8$, therefore $a = 4$.

7. Recognising Equations, Expressions and Different Types of Equations

i. Quadratic expression in the expanded quadratic form.

ii. Linear Equation

iii. Quadratic equation in the expanded factorised form.

iv. Formula for the area of a circle.

v. Exponential equation as shown by a variable power (4^x).

vi. Linear expression, as there are no powers, and an unknown variable.

vii. Quadratic equation in the expanded quadratic form.

viii. A linear inequality, shown by the greater than '>' sign. There is one unknown variable with no powers, so it can be solved.

ix. This fraction can be simplified to $\frac{x-4}{x}$ when we factorise and simplify.

x. A simple expression.

xi. A linear expression.

xii. A solved equation for the area of a circle.

xiii. A quadratic expression in the factorised form. It shows x-intercepts of -5 and $+4$

xiv. A solvable, exponential inequality.

xv. The addition of two fractions.

xvi. A linear equation, which can be simplified.

xvii. A quadratic equation that can be simplified further.

xviii. A solvable exponential equation.

xix. A solvable equation.

Section Two

1. Working through Word Equations

1.

a. The variables are Pat's current age and Tom's current age. We will call them p and t respectively.

b. The first part tells us that Pat is 12 years younger than Tom. Writing this in maths we get $\mathrm{p} = \mathrm{t} - 12$ or $\mathrm{t} = \mathrm{p} + 12$.

c. The second part tells us that $\mathrm{t} - 6 = 2(\mathrm{p} - 6)$

d. Substituting the first equation into the second gives p + 12 - 6 = 2(p - 6). This gives p = 18.

e. Substituting p = 18 into the first equation gives t = 30.

f. Therefore Pat is currently 18 and Tom is currently 30.

2.

a. The variables are the number of hours that you and your friend work. Let the hours that you work be x and the number of hours your friend works be y.

b. The equation for this is 15x + 20y = 750.

c. The equation for this is x = 2y.

d. Substituting c) into b) gives 30y + 20y = 750. These numbers are quite big but we can divide everything by 5 to make it easier. This gives 10y = 150 so y = 15 and therefore x = 30.

e. In total, you work 30 hours. At $15 an hour, you make $450. Your friend works 15 hours at $20 an hour and they make $300. This makes $750 in total, which is how much was needed.

3.

a. The variables are the time it takes for Alice and Max to finish the race. Let Alice's time be a and Max's time be b.

b. We are told that the difference in velocity is 1, so the equation for this is $\frac{200}{\mathrm{a}} - \frac{200}{\mathrm{b}} = 1$ because Alice finishes the race first.

c. The difference between Alice and Max is 10 seconds. This means that Max's time is 10 seconds more than Alice's, so if we subtract Alice's time from Max's we will get 10 seconds. This is the equation $\mathrm{b} - \mathrm{a} = 10$.

d. Making a substitution we get $\frac{200}{a} - \frac{200}{10+a} = 1$

When we subtract these fractions, we end up with a quadratic equation: $a^2 + 10a - 2000 = 0$, which factorises to $(a + 50)(a - 40) = 0$. Only one of these values makes sense because time can't really be negative. So $a = 40$ and this means that $b = 50$.

e. It takes Alice 40 seconds to finish the race and it takes Max 50 seconds to finish the race.

4.

a. Let a be the total number of adults and let s be the total number of students.

b. The equations are: $15a + 10s = 350 \quad a + s = 30$
Making a substitution for a, we get $15(30 - s) + 10s = 350$. We can divide everything by 5 to get $3(30 - s) + 2s = 70$, which means we get $s = 20$ so $a = 10$.

c. The variables in the second simultaneous equation are the number of people in group one, g_1 and the number of people in group two, g_2.

d. We make two equations;
$\frac{3}{4} g_1 + \frac{1}{2} g_2 = 20$ and $\frac{1}{4} g_1 + \frac{1}{2} g_2 = 10$

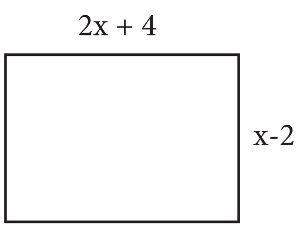
e. We rearrange the first equation for $\frac{1}{2} g_2 = 20 - \frac{3}{4} g_1$ and substitute this into the second equation to get $\frac{1}{4} g_1 + (20 - \frac{3}{4} g_1) = 10$ then we rearrange and find $g_1 = 20$.

f. We are told that three quarters of the first group are students, which means a quarter are adults, as seen in the second equation from d., which means there are 5 adults in group 1.

2. Mixed Problem Solving Questions

1.

a.



b. The formula for the area of a rectangle is A = width $\times$ length. The width in this case is $x - 2$ and the length is $2x + 4$. So to get the area we simply multiply the sides of the rectangle together. After simplifying we get $2x^2 - 8$ as the expression for the area.

c. We are told that the area is 42, so we set the expression equal to this to get $2x^2 - 8 = 42$. We are going to solve this directly by moving the constants all to one side to get $2x^2 = 50$. Then divide by 2 to get $x^2 = 25$, which is a perfect square. Taking the square root of both sides gives $x = -5, 5$.

d. The length and area must be positive, so only $x = 5$ is valid.

2.

a. We know that 25 is a power of 5, so we can rewrite the left-hand side as $5^{-(2x+1)+2} = 5^{3x^2}$.

b. This means we get the quadratic equation $3x^2 = -2x + 1$, which we can rearrange to get $3x^2 + 2x - 1 = 0$.

c. We need to use grouping, which gives us $(3x - 1)(x + 1) = 0$, and therefore the solutions are $x = -1, \frac{1}{3}$.

3.

a. The formula for the area of a right-angle triangle is $A = \frac{1}{2}\ \text{base} \times \text{height}$.

b. This means the base is $4x + 2$ and the height is $2x - 1$.

c. Substituting this into our expression for the area we get $A = \frac{1}{2}(4x + 2)(2x - 1)$.

d. $15 = \frac{1}{2}(4x + 2)(2x - 1)$.

e. Expand the brackets to get $15 = \frac{1}{2}(8x^2 - 2)$. Rearranging this gives $8x^2 = 32$, and so $x = -2$ or 2. $x = 2$ is the correct answer for this question because we can't have a negative length.

4.

a. $A = 3x^2 + 6x - 18$
$B = x^2 + 2x - 6$

b. Equation A is 3 times B.

c. $A = 3B$.

5.

a. The x-axis is measuring either the distance that the rocket has travelled, or the time of its flight.

b. $-x^2 + 8x = y$ the value of a is -1.

c. $-x^2 + 8x > 15$. Rearranging and factorising gives us $0 > (x - 3)(x - 5)$, which means that $x > 3$ and $x < 5$.

d. The rocket is above 15 m high between the distances or times of 3 and 5 metres or seconds.

6.

a. The missing sides are $x + 1$ and $4x - 1$.

b. The expression is all of the sides added up together. So, we have $P = 12x + 4$.

c. The equation we get is $12x + 4 = 64$. Rearranging this we find that x is 5.

7.

a. We rearrange the inequation to get $\frac{4}{x\text{-}1} > \frac{2}{x}$, by moving the terms that have x in their denominator.

b. We can then cross multiply to get $4x > 2x - 2$. Rearranging this we get $x > -1$.

c. Because x has to be positive, $x > 0$. Also, in the original inequation, we have two denominators, x - 1 and x. These can't equal zero because we can't divide things by zero. So, $x > 0$ and x can't be 1.

8.

a.

b. The ratio we get is $\frac{3x^2 + 13x + 10}{x^2 + 3x + 2} = 4$

c. We can factorise the top and the bottom to give $\frac{(3x + 10)(x + 1)}{(x + 2)(x + 1)} = 4,$ which simplifies to $\frac{3x + 10}{x + 2} = 4$. We can multiply by the denominator of the fraction to get the linear equation $3x + 10 = 4x + 8,$ so $x = 2$.

d. Substitute $x = 2$ into the two area equations to get the areas: large rectangle = 48, and small rectangle $= 12$.

9.

a. When four molecules are fixed, c $= 4$. The expression is $4^{n\text{-}4}$

b. $4^{n-4} > 250$.

c. For n = 1, 4^{-3} is less than 250.
For n = 2, 4^{-2} is less than 250.
For n = 3, 4^{-1} is less than 250.
For n = 4, 4^{0} is less than 250.
For n = 5, 4^{1} is less than 250.
For n = 6, 4^{2} is less than 250.
For n = 7, 4^{3} is less than 250.
For n = 8, 4^{4} is more than 250.

d. When n is more than 7, our DNA molecules have more than 250 possible combinations.

10.

a. The volume of the bottom part of the cone is the large cone minus the smaller cone. The volume of the large cone is $V = 5\pi {r_1}^2$ and the volume of the smaller cone is $V = \frac{5}{3}\pi {r_2}^2$ so the volume of the bottom part is $V_b = 5\pi {r_1}^2 - \frac{5}{3}\pi {r_2}^2 = 5\pi({r_1}^2 - \frac{1}{3}{r_2}^2)$.

b. If we draw the radius and heights of the cones as two right-angle triangles, we notice that these triangles must be similar, which means the ratio of their height and radius are equal. This gives us $\frac{15}{r^1} = \frac{5}{r^2}$ which means $r_2 = \frac{1}{3} r_1$.

c. We substitute this into our equation for the volume and simplify to get $V = 5\pi(\frac{26}{27}\ {r_1}^2)$.

3. Mixed Practice

i. $x = 0$ or $\frac{3}{2}$

ii. $x = 8$

iii. We can divide both sides by b. $a = -\frac{1}{3}$

iv. $x = 0$

v. $x = 1$

vi. $x = \frac{1}{2}, -4$

vii. $x = 4$

viii. $x = -1, 3$

ix. $x = -2, 2$

x. $x =$ -$\frac{1}{2}, 4$

Section Three

Exam

Question One

a. The equation is $6x + 6 = -12$ so rearranging gives $6x = -18$ so $x = -3$. She is thinking of the number negative 3.

b. We factorise by grouping to get $(4x + 1)(x - 2) = 0$ so $x = \frac{-1}{4}, 2$

c. First we expand A to get $2n^2 - 2n - 6$ and B to get $n^2 - n - 3$ so $A = 2B$

d. $x = 2$

e. r $= \sqrt{\frac{8}{7}A}$

Question Two

a.

i. The expression is $3x^2 - 5x - 2$

ii. The side lengths $3x + 1$ and $x - 2$ cannot be negative, which means that both of these terms need to be strictly positive. We can write an inequality for the smallest side being larger than 0 which is $x - 2 > 0$ which means that $x > 2$. As long as x is larger than 2 we will have a positive area.

b. Given what p is, we can write $E = \frac{1}{2}(mv)v$ which simplifies to $E = \frac{1}{2}mv^2$.

c. $y =$ -6.

d. We can't write this is a single base, so we have to do some rearrangement. We divide both sides by 2 to get $3^{n-2} < 27$ and now we can rewrite as a single base. $3^{n-2} < 3^3$ so $n - 2 < 3$ so $n < 5$. Since n is an integer, it can be less than 0, so this is the final answer.

e. Divide by 4 to get $27 = r^3$. We can then take the cubed root of both sides to get $r = 3$. So the rate they spread their influence is 3.

Question Three

a.

  i. The other side is given by the equation $x - 2$.

  ii. We form the equation $x^2 - 3x + 2 = 56$. Rearranging this we get $x^2 - 3x - 54 = 0$. Then we factorise to get $(x - 9)(x + 6) = 0$ and so we get $x = 9$ and -6. Only one of these values makes sense to get a positive area so $x = 9$.

b. Factorise this to get $(x-2)(x - 3) = 0$ and therefore $x = 2{,}3$.

c. The equation for the perimeter is $8x - 3 + 2x + 1 + 3x - 2 + x + 5 + c = 20x + 2$. Simplifying this gives $14x + 1 + c = 20x + 2$. Rearranging for c in terms of x gives $c = 6x + 1$.

d. If the quadratic has only one solution, then it is of the form $0 = (x+a)^2 = x^2 + 2ax + a^2$. Comparing coefficients gives $2a = 8$, so $a = 4$.

   So, $c = 16$. Putting this into the equation and solving for x gives $x =$ -4.

e. The volume of the larger cylinder is given by $V_{large} = \pi 10R^2$ and the volume of the smaller cylinder is $V_{small} = \pi 10r^2$.

   We are told that $V_{large} = 4V_{small}$ so $\pi 10R^2 = \pi 40r^2$.

   We want R in terms of r, so we divide both sides by $\pi 10$ to get $R^2 = 4r^2$. Then we take the square root of both sides to get $R = \sqrt{4r^2}$ which simplifies to $R = 2r$.

