Algebra
ANSWERS

The Language of Maths

The Language of Maths

STOP AND CHECK (PAGE 7)

- We need to be able to use mathematical language to clearly communicate ideas and connect the world around us.
- An expression contains variables and operations to describe something. An equation is similar but has an equal (=) sign.
- 'In terms of' means that a variable has been made the subject of the equation, i.e. is on one side of the equals sign on its own.

Algebra Basics

STOP AND CHECK (PAGE 11)

- Expand the brackets, gather all like terms on one side of the equation, isolate x, rearrange to make x the subject of the equation and then simplify.

Expanding and Factorising Quadratics

STOP AND CHECK (PAGE 14)

- $(x - 1)(x - 2)$
- $(x - 5)(x - 7)$

Factorising With a Coefficient Bigger than 1

STOP AND CHECK (PAGE 15)

- $(2x - 1)(x + 2)$
- $4(x - 5)(x + 4)$
    - Notice for this one you may have gotten $(4x - 20)(x + 4)$. That's great, but you should be looking to simplify by the smallest factor. Here, this is noticing that 4 is a factor of 20 so we pull it out the front and reduce the variables inside the brackets as we have above.

Expanding Cubics

STOP AND CHECK (PAGE 17)

- $x^3 + 4x^2 - 11x - 30$
- $x^3 + 7x^2 + 8x - 16$

Expanding and Factorising

QUICK QUESTIONS (PAGE 17)

- The garden is a rectangular shape so we know the perimeter (the distance around the outside) is 2 times the length plus 2 times the height of the garden. We know the length of the garden is x metres long. We do not know the height of the garden, but we can create an expression for this using the area. We will use h to represent height.

  $$\text{Area of a rectangle} = \text{Length} \times \text{Height}$$

  So, for Vanessa's garden with an area of 20m² we have $20 = xh$. We can re-arrange this to make h the subject"

  $$h = \frac{20}{x}$$

  Now we have an expression for height, we can use this to create our expression for perimeter:

$$\text{Perimeter} = 2x + 2h$$
$$\text{Perimeter} = 2x + 2\left(\frac{20}{x}\right)$$

This simplifies to:

$$\text{Perimeter} = 2x + \frac{40}{x}$$

- We need to find the dimensions so we need to find what numbers h and *x* are. We have been told the perimeter is 18m so we can put this value in our expression for the perimeter:

$$18 = 2x + \frac{40}{x}$$

We need to solve this equation for *x*.

$$18x = 2x^2 + 40$$
$$2x^2 - 18x + 40 = 0$$
$$2(x^2 - 9x + 20) = 0$$
$$2(x - 5)(x - 4) = 0$$
$$x = 5 \text{ or } 4$$

We know that the length must be 5m or 4m and therefore the dimensions of the garden are 4m by 5m.

Algebraic Fractions

Squashing Down Fractions to their Simplest Form

STOP AND CHECK (PAGE 20)

- $\frac{8x}{3}$
- $\frac{24}{5x} - \frac{6}{5}$
    - Note: This one could also have been left in the form $\frac{24 - 6x}{5x}$

Simplifying Fractions with Higher Powers

STOP AND CHECK (PAGE 21)

- A common factor is a number that divides both of two other numbers.
- Divide the numbers by a common factor then subtract the bottom powers from the top for each letter.

How to Add, Subtract and Multiply Fractions

STOP AND CHECK (PAGE 24)

- Draw an upside-down picnic table.
- Multiply each half of the fraction straight across. In other words, multiply straight across the top (i.e. the numerators), then do the same thing for the bottom (i.e. the denominators).

Rearranging Algebraic Fractions

STOP AND CHECK (PAGE 27)

- $x = -\frac{9}{22}$
- $x = 0$

Rearranging Algebraic Fractions

QUICK QUESTIONS (PAGE 27)

- $y = \frac{5x}{4(x-3)}$
    - $y = \frac{5x}{4x-12}$
- $y = \frac{6}{3x} - \frac{4x+x^2}{3x}$
- $y = \frac{x^2+4x+6}{3x}$

Exponents and Logarithms

The Basics of Logarithms and Exponents

STOP AND CHECK (PAGE 30)

- Base to the bottom, y in the brackets.

Adding, Subtracting, Dividing and Multiplying Logarithms

STOP AND CHECK (PAGE 32)

- $\log_3(50) - \log_5(8)$
- $\log\left(\frac{x^3}{4y^2}\right)$

Solving Logarithmic Equations

STOP AND CHECK (PAGE 34)

- m = 2
- m = 9

Logarithmic Word Problems

STOP AND CHECK (PAGE 37)

- $587.12
- He will have to wait 4.9 years.

Exponents and Logarithms

QUICK QUESTIONS (PAGE 37)

- We need to make the general function specific to the context we are given. *A* will represent the initial price of $2499. *r* will represent the rate of price

decrease every year, this decimal is 0.88 since it decreases by 12% every year. This will find the remaining 88% of the price every year. So, our equation is:

$$P = 2499 \times (0.88^t)$$

The halved price of the laptop is $1249.50. We put this into our equation and solve for time:

$$1249.50 = 2499 \times (0.88^t)$$
$$0.5 = 0.88^t$$
$$\log(0.5) = \log(0.88^t)$$
$$\log(0.5) = t \log(0.88)$$
$$t = \frac{log(0.5)}{log(0.88)}$$
$$t = 5.42$$

It takes about 5 and a half years to decrease by half in price.

- We have to use substitution as we can't solve a logarithm when the square is not inside of the brackets. Lets us $a = \log_4 x$ for our substitution:

$$6a^2 + 2a - 4 = 0$$

We can now solve this by factorising:

$$2(3a^2 + a - 2) = 0$$
$$2(3a^2 + 3a - 2a - 2) = 0$$
$$2(3a(a + 1) - 2(a + 1)) = 0$$
$$2(3a - 2)(a - 1) = 0$$
$$a = -1, \frac{2}{3}$$

Now that we know the solutions, we can go back to the log we substituted in for a:

$$a = \log_4(x) = -1, \frac{2}{3}$$

Using log rules on $\log_4(x) = -1$, we get $x = 4^{-1}$ which becomes ¼. Similarly, $\log_4(x) = \frac{2}{3}$ gives $x = 4^{⅔}$ which becomes 2.52.

Quadratics and Polynomials

Solving Quadratics and Completing the Square

STOP AND CHECK (PAGE 40)

- $x = -2 \pm \sqrt{11}$
- $x = 3 \pm \sqrt{24}$

The Quadratic Formula

STOP AND CHECK (PAGE 42)

- $x = \frac{4 \pm \sqrt{160}}{6}$
    - So, $x = 2.77$ or -1.44
- $x = \frac{-12 \pm \sqrt{244}}{10}$
    - So, $x = 0.362, -2.76$

The Discriminant and the Magical Nature of Roots

STOP AND CHECK (PAGE 45)

- $k = 2$
- $k > 2$

Quadratics and Polynomials

STOP AND CHECK (PAGE 46)

- First, let's solve $p^2 + p - 56 = 0$:
    - $(p - 7)(p + 8)$
    - $p = -8, 7$

  Now, let's solve $4p^2 + p - 14 = 0$:
    - $4p^2 + 8p - 7p - 14 = 0$

- $4p(p + 2) - 7(p + 2) = 0$
- $(4p - 7)(p + 2) = 0$
- $p = -2, \frac{7}{4}$

-8 is 4 times bigger than -2, and 7 is 4 times greater than $\frac{7}{4}$.

- For the equation to have real rational solutions, $b^2 - 4ac$ has to be greater than or equal to 0. In this equation:
  - $a = 2$
  - $b = 4m$
  - $c = 2m^2 + 3m - 11$

  We put these values into the discriminant and solve for m:
  - $(4m)^2 - 4(2)(2m^2 + 3m - 11) \geq 0$
  - $16m^2 - (16m^2 + 24m - 88) \geq 0$
  - $-24m + 88 \geq 0$
  - $88 \geq 24m$
  - $\frac{11}{3} \geq m$