Probability

ANSWERS

Probability Trees and Two-Way Tables

Basic Proportions and Probabilities

STOP AND CHECK (PAGE 7)

- Favourable outcomes: the result we are looking for or the thing we are looking at.
- A probability of 1 means it will definitely happen.

Growing Probability Trees

STOP AND CHECK (PAGE 10)

- P of choosing a skittle after choosing an M&M or P(skittle after M&M) is $\frac{10}{59}$
- The denominator is the total number of possible outcomes. Now that you have eaten one lolly, there is one less lolly in the bowl, so the denominator decreases. For example, if there were 60 lollies in the bowl but after eating one there would be only 59 left.

Using Probability Trees

STOP AND CHECK (PAGE 13)

- Finds the probability of multiple events occurring.
- P(choosing no skittles) = $\frac{5}{6} \times \frac{49}{59}$
    - P(choosing no skittles) = $\frac{245}{354}$

Two-Way Tables

STOP AND CHECK (PAGE 15)

- Two different groups of people compared against some sort of event.
- Two-way tables with proportions are different to tables with amounts as you need to multiply each proportion for each event together, rather than using the amounts to create a proportion.

Probability Trees and Two-Way Tables

QUICK QUESTIONS (PAGE 15)

- The probability of choosing a red lolly and lemon lolly in any order has two pathways:
    - P(Choosing red, then lemon): $\frac{7}{15} \times \frac{3}{14} = \frac{21}{210}$ or $\frac{1}{10}$
    - P(Choosing lemon, then red): $\frac{3}{15} \times \frac{7}{14} = \frac{21}{210}$ or $\frac{1}{10}$

  Therefore, the total probability is:
    - $\frac{1}{10} + \frac{1}{10} = \frac{2}{10}$ or 0.2
- The probability of choosing two different coloured jet planes has six pathways:
    - P(Red then purple): $\frac{7}{15} \times \frac{5}{14}$
    - P(Red then lemon): $\frac{7}{15} \times \frac{3}{14}$
    - P(Purple then red): $\frac{5}{15} \times \frac{7}{14}$
    - P(Purple then lemon): $\frac{5}{15} \times \frac{3}{14}$
    - P(Lemon then red): $\frac{3}{15} \times \frac{7}{14}$
    - P(Lemon then purple): $\frac{3}{15} \times \frac{5}{14}$

  We could multiply these out then add them all together to find the answer. However, there is a simpler way. We can identify the three pathways that we **don't** want, and take this away from 1:
    - P(Red twice): $\frac{7}{15} \times \frac{6}{14} = \frac{1}{5}$
    - P(Purple twice): $\frac{5}{15} \times \frac{4}{14} = \frac{2}{21}$
    - P(Lemon twice): $\frac{3}{15} \times \frac{2}{14} = \frac{1}{35}$

$$1 - \left(\frac{1}{5} + \frac{2}{21} + \frac{1}{35}\right) = \frac{142}{210} \text{ or } 0.68$$

The probability of choosing at least one lemon jet plane has five different pathways:

- P(Red then lemon): $\frac{7}{15} \times \frac{3}{14}$
- P(Purple then lemon): $\frac{5}{15} \times \frac{3}{14}$
- P(Lemon then red): $\frac{3}{15} \times \frac{7}{14}$
- P(Lemon then purple): $\frac{3}{15} \times \frac{5}{14}$
- P(Lemon twice): $\frac{3}{15} \times \frac{2}{14}$

So the total probability is:

$$\left(\frac{7}{15} \times \frac{3}{14}\right) + \left(\frac{5}{15} \times \frac{3}{14}\right) + \left(\frac{3}{15} \times \frac{7}{14}\right) + \left(\frac{3}{15} \times \frac{5}{14}\right) + \left(\frac{3}{15} \times \frac{2}{14}\right) = \frac{13}{35} \text{ or } 0.37$$

- The colours that have the highest probability of being pulled out when two jet planes are chosen are two red jet planes.
  - P(Red twice): $\frac{7}{15} \times \frac{6}{14} = \frac{1}{5}$

  This probability is the highest of all possible pathways.

Normal Distribution

The Normal and Standard Normal Distribution

STOP AND CHECK (PAGE 21)

- By turning the normal distribution into the standard normal we are able to find out where our data fits into the standard bell curve, shown by the z-score. We can then use this to calculate our probabilities.
- The mean becomes zero on a standard bell curve.
- The *x* value becomes the original random variable we are trying to find the probability of occurring.

Finding Distributed Probabilities

STOP AND CHECK (PAGE 26)

- To find a 'less than' probability using a calculator, we will first need to find the Ncd function on our calculator STATS MENU (2) > DIST (F5) > NORM (F1) > Ncd (F2)
- Then enter the following values:
    - **Lower:** make this a big negative number e.g. -1000000
    - **Upper:** this will be the *x* value (for example, less than 200g, so 200 would be entered here).
    - **σ:** is the standard deviation.
    - **μ:** is the mean.
- The key to working out 'less than' probabilities is to make sure that the lower value is a really big negative number.
- The *x* value is the original random variable that we are trying to find the probability of, whereas the z-score helps us find the position of our variable on the standard bell curve.

Inverse Normal

STOP AND CHECK (PAGE 28)

- If we are missing the standard deviation σ or mean μ, we can rearrange the standard normal formula to find either of them:

$$Z = \frac{x - \mu}{\sigma}$$

- However, before we can use this formula, we need to find the z-score. We do this by using the calculator's inverse function for a standard normal distribution. This means that the mean will be 0 and the standard deviation will be 1.
- Once we have this, we can rearrange the formula and use the values we do have to find either the mean or standard deviation.
- The question will use inverse normal if a probability is given and we are asked to find a value for *x*.

Normal Distribution

QUICK QUESTIONS (PAGE 29)

- $P(x < 62) = ½$ or 0.50

  This is because 62g sits at the halfway point of the bell curve. Alternatively, the following values could be entered into the calculator:
  - **Lower:** -1000000
  - **Upper:** 62
  - **σ:** 3
  - **μ:** 62

  This will also give a probability of 0.5.
- $P(60 > x > 80) = P(x < 60) + P(x > 80)$

  So these will be calculated separately using the calculator, using the following parameters:
  - For $P(x < 60)$:
  - **Lower:** -1000000
  - **Upper:** 60
  - **σ:** 3
  - **μ:** 62
  - For $P(x > 80)$:
  - **Lower:** 80
  - **Upper:** 1000000
  - **σ:** 3
  - **μ:** 62

  Therefore, $P(60 > x > 80) = P(x < 60) + P(x > 80)$
  - $P(60 > x > 80) = 0.2525$

  E(X) = number of trials×probability
  - E(X) = 20 x 0.2525
  - E(X) = 5.05

  Of the box of 20 chocolates, 5 of them are likely to cause grief for Vinash and Jara.
- This is an inverse normal question. By entering the following parameters into the calculator:
  - **Tail:** right (because the chocolates are medium-sized or larger)
  - **Area:** 0.8
  - **σ:** 3
  - **μ:** 62

x = 59.48g, so the weight of a medium-sized chocolate is 59.48g.

Conditional Probabilities and Expected Value

Expected Value and Theoretical Probability

STOP AND CHECK (PAGE 31)

- Theoretical probability is the calculated chance of something happening and exists where probabilities are random and there are no biases.
- The expected value is calculated by multiplying the number of trials by the theoretical probability.
- $E(X) = n \times p$

Conditional Probability

STOP AND CHECK (PAGE 34)

- P(child wins) = $\frac{25}{100}$
  - $\frac{1}{4}$or 0.25.
- P(win | child) = $\frac{25}{40}$
  - $\frac{5}{8}$ or 0.625

Conditional Probability with Trees

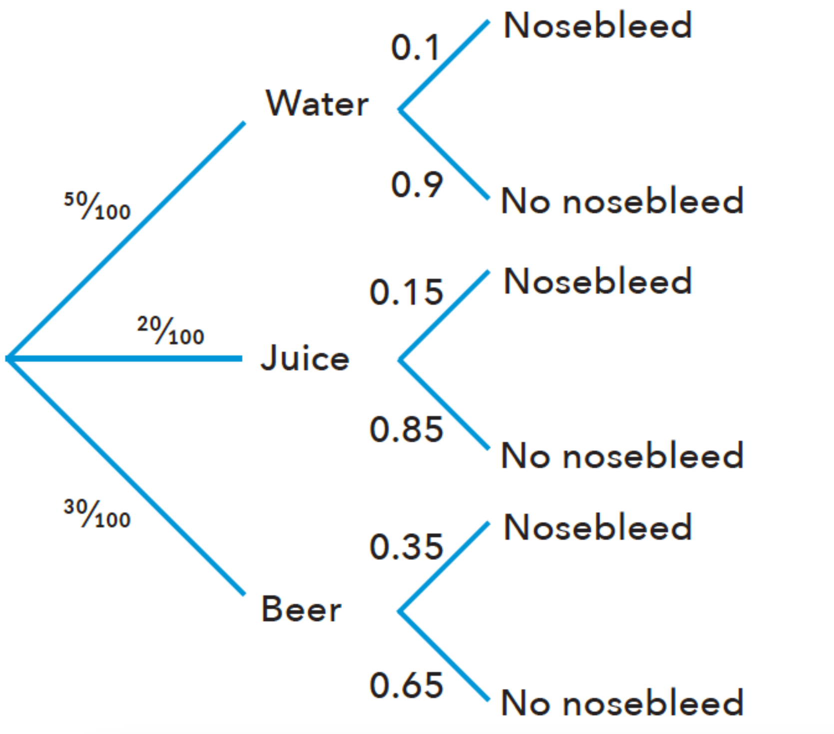
STOP AND CHECK (PAGE 35)

- To work out the probability that there was no lion, we would multiply out two branches:
  - P(rain and no lion) = 0.4×0.9
    P(rain and no lion) = 0.36
  - P(fine and no lion) = 0.6×0.25
    P(fine and no lion) = 0.15

- Dependent events mean that they rely on each other when one event only happens because another event happens.

Conditional Probabilities and Expected Value

QUICK QUESTIONS (PAGE 35)



- P(Nosebleed): $(0.5\times0.1) + (0.2\times0.15) + (0.3\times0.35) = 0.185$
- P(Beer | nosebleed): $(0.3\times0.35)\div0.185 = 0.5676$

Risk and Relative Risk

Risk

STOP AND CHECK (PAGE 37)

- Risk is simply a probability. The reason it is called risk is that it often relates to health conditions, disease and medical stuff.

- Absolute risk refers to risk or probability that is by itself and is not being related to any other probability or risk.

Relative Risk

STOP AND CHECK (PAGE 39)

- Relative risk has that name because we are relating two absolute risks with each other.
- When calculating relative risk we are not working out a probability, we are working out a comparison. Therefore, for the number to make sense in context we say, 'x times more likely'.

Relative Risk and the Normal Distribution

STOP AND CHECK (PAGE 41)

- We are able to make a claim when the relative risk is more than 2. This means that the risk in one group is twice as likely as the others. A relative risk of less than 2 is not strong enough evidence to back the claim.
- We want to prove or disprove claims to see whether the group is telling the truth, and this makes us more responsible consumers.

Risk and Relative Risk

QUICK QUESTIONS (PAGE 43)

- P(flu) = $\frac{17}{110}$ or 0.1545
- P(oranges | flu) = $\frac{12}{17}$ or 0.7059
- First, calculate the relative risks:
  - P(flu | oranges) = $\frac{12}{64}$
  - P(flu | no oranges) = $\frac{5}{56}$
  - The relative risk of flu for those eating oranges compared to no oranges: $\frac{12}{64} \div \frac{5}{56} = 2.1$

  Children are 2.1 times as likely to catch the flu when they have oranges. Therefore, these calculations show that the claim is correct "that children who got oranges are at least twice as likely to catch the flu as children who didn't get oranges".