PROBABILITY CONCEPTS
ANSWERS

Basic Probabilities

Basic Probability and the Language of Probability
STOP AND CHECK (PAGE 8)

- Probabilities are measured between 0 and 1.
- A fraction is:

$$\frac{outcome\ of\ interest}{total\ number\ of\ outcomes}$$

  A proportion, however, is a decimal, so is a single number.
- Fractions are often better as they are more exact.
- ∪ stands for the union of events, or all the outcomes that fit into one or both categories.
- ∩ stands for the intersection of events, or only the outcomes that fit into both categories.

The Union of Events
STOP AND CHECK (PAGE 9)

- The union of events is all the outcomes that fit into one or both categories.
- To find it you add the probabilities of event A and event B, then minus the intersection (the probability of both happening).

Complementary Events

STOP AND CHECK (PAGE 11)

- For example, the circle is yellow, or it is not yellow. When you roll a die, you get a 6 or you don't get a 6.
- All complementary (or separate) probabilities of an event add to 1.

Independent Events

STOP AND CHECK (PAGE 14)

- For example, you get heads when you flip a coin and you win the lottery. You roll a 2 and it is cloudy.

Mutually Exclusive Events

STOP AND CHECK (PAGE 11)

- For example, you turn left, or you turn right. It is cloudy, or it is not cloudy.
- Events are mutually exclusive when they cannot happen at the same time.

Basic Probabilities

QUICK QUESTIONS (PAGE 15)

- Lying and telling the truth are complementary events as you can only do one or the other, so their probabilities should add up to 100%. P(Edmund is telling the truth) = 100 – 66.5 = 33.5%
- The quickest way is to use complementary events. P(tells the truth at least once) = 1 – P(does not tell the truth at all). After converting to decimals, P(does not tell the truth at all) = 0.665^4 = 0.1956. 1 – 0.1956 = 0.8044.
- The events "The car was manufactured in Germany" and "The car is a used vehicle" are not mutually exclusive because mutually exclusive events are those that cannot happen at the same time. There is some overlap between these events, with (0.639 x 0.803 = 0.513) 51.3% of all the cars in the district being imported German cars. As the events can happen at the same time

(there are used cars in the district from Germany), they cannot be mutually exclusive.

Data Displays

Two-way Venn Diagrams

STOP AND CHECK (PAGE 19)

- Be careful not to use Venn diagrams when events are mutually exclusive and make sure to calculate the intersection of events before you start to write in numbers.
- For Venn diagrams use the union formula: $P(A \cap B) = P(A) + P(B) - P(A \cap B)$

Three-way Venn Diagrams

STOP AND CHECK (PAGE 23)

- The first thing to do with these is whatever isn't in the circles and whatever is in all three.

Two-way Tables

STOP AND CHECK (PAGE 24)

- To find a probability from a two-way table you can either read it straight off the table (if it has percentages or proportions) or you divide the number in that category by the total (if it has raw data).
- Two-way tables can show the relationships between two events, using raw data, percentages or proportions.

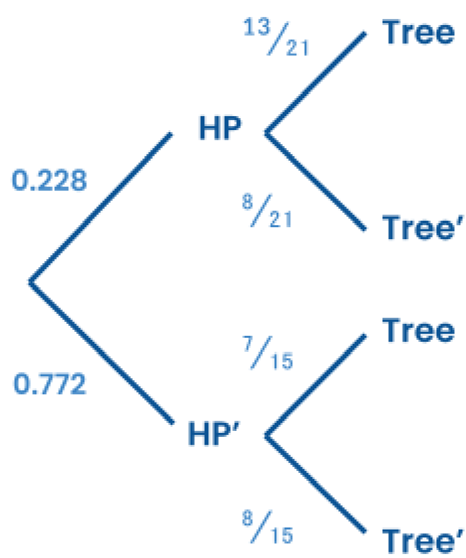
Probability Trees

STOP AND CHECK (PAGE 29)

- Pairs of branches add to 1.
- Probability trees are good at showing conditional events.

Deduction in Tree Diagrams

STOP AND CHECK (PAGE 30)



- For example, a box contains 5 green marbles and 3 purple marbles. Two marbles are drawn without replacement.

Data Displays

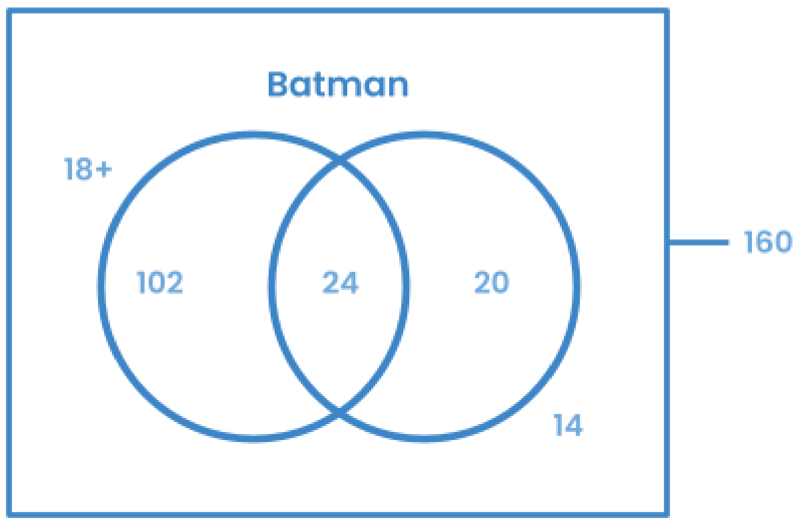
QUICK QUESTIONS (PAGE 30)

- Start by filling in information from the question. The percentages are written as decimals to make any calculations easier. 0.228 of all road cones were stolen around Hagley Park (HP) so the proportion of those not stolen around Hagley Park (HP’) must be 1 – 0.228 = 0.772 as branches should add up to 1. Then start a new set of branches. $\frac{13}{21}$ of those stolen around Hagley Park reappeared in trees (Tree) so the proportion of those stolen around Hagley Park that didn’t reappear in trees (Tree’) is:

$1 - \frac{13}{21} = \frac{8}{21}$

$\frac{7}{15}$ of those not stolen from Hagley Park reappeared in trees (Tree) so the proportion of cones not stolen from around Hagley Park that didn't reappear in trees (Tree') is:

$1 - \frac{7}{15} = \frac{8}{15}$



- Though it may not look like it, there are only two categories for this Venn diagram as everyone saw the Dark Knight trilogy in the study. The categories are over 18 and think Batman is the best superhero. 24 students are over 18 and think Batman is the best superhero, so this goes in the middle of the diagram. 20 students think Batman is the best superhero but are not over 18, so this goes in the circle on the right. 102 are over 18 but do not think Batman is the best superhero, so this goes in the circle on the left. These numbers should all add up to 160, and 160 – 102 – 24 – 20 = 14, so 14 students in the study are not over 18 and did not think Batman is the best superhero, so this goes outside the circles.

Conditional Probability and Risk

Conditional Probabilities

STOP AND CHECK (PAGE 33)

- P(child wins) = $\frac{25}{100}$ = 0.25
- P(win | child) = $\frac{25}{40}$ = 0.625

Conditional Probabilities With Trees

STOP AND CHECK (PAGE 36)

- To find the probability there was no lion you would use P(no lion | rain) + P(no lion | fine) as there are two ways this could happen.
- Dependent events are ones where the probability depends on the outcome of another event.

Risk and Relative Risk

STOP AND CHECK (PAGE 38)

- Risk is the probability of $\frac{a\ person}{group\ getting\ a\ problem}$, while relative risk looks at how likely it is for a person/group to get a problem compared to the probability for another person/group.

Conditional Probability and Risk

QUICK QUESTIONS (PAGE 39)

	Over 18	Not over 18	Total
P(Batman)	84	32	116
P(Batman')	1206	678	1884
Total	1290	710	2000

- The first step is to create a table, as shown above. There are 2000 students total and 1290 are over 18, so the number not over 18 is 2000 – 1290 = 710. The total that though Batman was the best is 116, so the number who don't think Batman is the best is 2000 – 116 = 1884. 32 thought Batman was the best and were not over 18 years old, so the number not over 18 who thought Batman wasn't the best is 710 – 32 = 678. Of those who thought Batman was the best, 116 – 32 = 84 were over 18. Of those who didn't think Batman was the best, 1884 – 678 = 1206 were over 18. Now the actual question can be answered:

  P(Batman not the best and over 18) = $\frac{1206}{2000}$ = 0.603.

- Firstly, you need the probability that one student was over 18 given that they thought Batman was the best.

  P(over 18 | Batman the best) = $\frac{84}{116}$ = 0.724

  This probability is out of 116 as it is dependent on the student thinking Batman is the best. To get the probability of two randomly selected students being like this, the probability just needs to be squared: 0.724^2 = 0.524.

- Firstly, the risk for cones being stolen each year needs to be calculated:

  P(cones stolen in 2014) = $\frac{724}{10511}$= 0.0689
  P(cones stolen in 2015) = $\frac{807}{18612}$ = 0.0433
  P(cones stolen in 2016) = $\frac{639}{15339}$ = 0.0417

  The highest proportion of cones stolen was in 2014, so 2014 had the greatest overall risk of cones being stolen in Christchurch.

- The risks calculated above are only estimates of the true overall risk because they only consider the number of cones that were reported as stolen. There are likely to be more cones that went missing but were not reported as stolen for a variety of reasons, such as mistakes made in the number recorded. As a result, the true overall risk of cones being stolen in Christchurch could be higher.

Experimental, Theoretical, and True Probability

Theoretical Probability

STOP AND CHECK (PAGE 42)

- For example, the theoretical probability of randomly picking a green ball out of a bag containing 4 green balls and 5 pinks.

Experimental Probability

STOP AND CHECK (PAGE 44)

- For example, the experimental probability of flipping tails when a coin has been flipped 80 times and the result was 34 tails.

Theoretical, Experimental and True Probability

STOP AND CHECK (PAGE 46)

- When comparing theoretical and experimental probabilities, you should discuss how to always expect some difference from variation, using more trials to reduce variation, and huge differences probably meaning that the trials are not random.
- You cannot be sure about true probabilities because you would have to have an infinite number of trials to get the exact answer, which is not possible.

Experimental, Theoretical and True Probability

QUICK QUESTIONS (PAGE 46)

- It would be helpful for the teacher to investigate the variability in the experimental probability of the number of turns it takes to reach the 5th stair. 30 students each playing the game is quite a small sample. This means that differences between theoretical and experimental variation might just be from natural variation, not cheating by some students. A larger sample size would reduce this natural variation, so the teacher can more accurately see whether students are cheating.

