Mechanical Systems
ANSWERS

Fundamentals

Period and Frequency
STOP AND CHECK (PAGE 18)

- In mechanics, a system is a collection of objects that we wish to describe the motion of. This system can be defined in terms of whatever you deem relevant. Forces that originate from outside of this system are called external forces.
- A scalar quantity only has a magnitude, which is how large the quantity is; the direction doesn't matter. Three examples are mass (kg), length (m) and time (s). A vector quantity has both a magnitude and a direction. Three examples are force (N), angular momentum (L) and velocity (ms^{-1}).
- Because 180° is π radians we can use the formulae:

$$x = \left(\frac{180}{\pi}\right)\theta$$

  And,

$$\theta = \left(\frac{180}{\pi}\right) x$$

  Where x is an angle in radians and θ is an angle in degrees. Although the formula is useful, it's best to understand where the radians measure comes from for circles then you will be able to convert degrees to radians and vice versa with ease!
- They are inverses of each other. A good way to think about frequency is how many times does a thing happen per second (like a revolution) and period as

being how long it takes a thing to happen (like a revolution). This is where the units Hz (s^{-1}) and s come from!

- Momentum is conserved whenever there are no external forces acting on a system, a system being a set of things that you are focusing on. These forces can be linear, as in the collisions you did last year, or rotational, in which case no external torques (a torque is a 'turning force') are applied and so angular momentum is conserved. This will be very important later.
- Yes! This is a fundamental law of the universe and it's incredible if you think about it. It means that energy cannot be created or destroyed, only transformed into different types of energy or transferred through interactions. This means that the total amount of energy before something happens and after it happens is the same, it may just be in different forms.
- No, just because energy in general is conserved doesn't mean that kinetic energy is conserved. Usually, some amount of kinetic energy will be transformed into heat due to friction.
- The Work-Energy Principle states that work is the change in kinetic energy of an object (or objects). Since changing the kinetic energy of an object requires acceleration, doing work involves a force. Work is essentially how much energy is required to do something. Its unit is the Joule.

The Fundamentals

QUICK QUESTIONS (PAGE 18)

- Let
    - $x_1 = 0$
    - $x_2 = 1.5m$
    - $m_1 = 63kg$
    - $m_2 = 70kg$

$$X_{com} = \frac{m_2 x_2}{m_1 + m_2}$$

$$X_{com} = \frac{(70)(1.5)}{70 + 63}$$

$$X_{com} = \frac{105}{133}$$

$$X_{com} = 0.8m \text{ from } x_1$$

Because we made X_1 Bo's location ($x_1 = 0$) the point of reference, the COM is 0.8m from Bo's Position.

- Ronnie and Bo are demonstrating Newton's First Law: "An object in motion will stay in motion, unless acted on by an external force"
- Converting radians to degrees:

$$3\pi \times \frac{180}{\pi} = 540^{\circ}$$

Translational Motion

Kinematics

STOP AND CHECK (PAGE 19)

- This funny lil triangle is the Greek letter delta and it means the **change in**. It's used to calculate how much something has changed we take its final value and subtract the initial value so: $\Delta x = x_{final} - x_{initial}$
- We use Kinematic equations when we have to calculate the motion of objects when we aren't given average velocity or acceleration.

Circular Motion

STOP AND CHECK (PAGE 20)

- The formula for the circumference of a circle is $C = 2\pi r$
- The symbol θ is the Greek letter theta and it is the typical symbol for an angle. In circular motion and angular mechanics, we always work in radians, so the unit is rad.
- The symbol ω is the Greek lowercase letter omega and it is the symbol for angular frequency, also known as angular speed. This is how many radians something rotates in one second, so the unit is $rads^{-1}$.
- In circular motion, the frequency refers to how many rotations the object does around its circular path in a second.

Centripetal Force

STOP AND CHECK (PAGE 23)

- A centripetal force is a centre-seeking force. It's a force that causes something to travel in a circular path by continuously changing the direction of the velocity vector to be at a tangent to the path. Therefore the direction of the centripetal force must always be towards the centre of the circular path.
- Linear and rotational velocities are connected by the radius of the circular path. When something is moving in a circle, every part of it moves through the same angle, so at different radii, the object will be travelling a different distance (the circumference) in the same amount of time. Therefore, the linear velocity of a particle at a given radius from the centre of rotation is given by $v = r\omega$, where r is the radius (in m) and ω is the angular velocity (in $rads^{-1}$).
- If the centripetal force suddenly disappears, the object will continue to travel in a straight line, that is, at a tangent to the circular path. If no forces are acting, this will be a constant velocity because there is no acceleration.

Gravity

STOP AND CHECK (PAGE 27)

- Newton's 3rd law applies to all forces. Suppose we have two objects with mass; this means that if object A attracts object B due to its gravity then object B also attracts object A.
- Gravity acts as a centripetal force when objects are in orbit, that is, the direction of the orbiting objects' velocity is changing all the time. If the horizontal velocity is sufficiently large then the object will fall around the earth. We call this orbiting.
- The force due to gravity is given by the equation, $F_g = \frac{GMm}{r^2}$. As the rocket gets further from earth, the force due to gravity will change because the radius is getting larger. In fact, the force due to gravity will drop off pretty quickly. We call this the inverse square law because the force is proportional to the inverse square of the radius.
- A geostationary orbit is when an object orbits with the same period (and frequency) as the rotation of the body that it orbits around. This is useful for things like communication satellites. This means it is always at the same point above the object it is orbiting.

Translational Motion

QUICK QUESTIONS (PAGE 27)

- This is because of Newton's third law: "Every action will have an equal and opposite reaction" This means that as the ship fires the proton gun, the propulsion force of the protons will be matched by an equal and opposite force backwards; recoil.
- At the bottom of the loops, the model's weight force will act opposite to the centripetal force that is keeping it in a circular motion (the lift force). In order to counteract this, and ensure the model stays in a circular motion, the lift force must be significantly larger than the weight force.
- The minimum speed will occur at the top of the loop. At this point, the entirety of the centripetal force will be provided by gravitational acceleration.

$$a_c = \frac{v^2}{r}$$

$$9.81 = \frac{v^2}{0.7}$$

$$\sqrt{0.7(9.81)} = v$$

$$2.62ms^{-1} = v$$

Rotational Motion

The Basics

STOP AND CHECK (PAGE 31)

- These are the rotational motion quantities. They are angle (θ) which is the same as distance in linear motion and is measured in radians; rotational velocity (ω) or angular frequency which is measured in radians per second, and angular acceleration (α) which is measured in radians per second per second ($rads^{-2}$). Angle is the angle about the centre of rotation that the object rotates; the rotational velocity is what angle it rotates through in a second and angular acceleration is that rate at which the rotational velocity changes.
- There are 2π radians in one rotation. In other words, there are 2π radians in 360°.

- This is true, the angular speed is the same for all points on a rigid body that is rotating.
- This is not true, the linear speed changes as you change the radius of rotation. The linear speed is proportional to the radius (the distance from the objects' centre of rotation). The greater the radius, the greater the linear speed, given that the rotational velocity remains the same.

Rotational Kinematics

STOP AND CHECK (PAGE 32)

- A good way to figure out which kinematic equation to use is to write down the quantities that you are given and which quantity you are trying to find out. Each kinematic equation only has 4 quantities and there are 5 rotational quantities in total; they have to give you 3 quantities and leave out a 4th one so you will be able to narrow down which equation to use.
- Angular acceleration is how much the angular velocity changes in a given time; it is the rate of change of angular velocity.

Rotational Inertia

STOP AND CHECK (PAGE 34)

- If there is a net force, we will have an acceleration of the centre of mass of an object.
- If there is a net torque, there will be a rotation about the centre of mass of an object.
- I would probably describe rotational inertia as being how difficult it is to rotate something.
- Mass of an object and the distribution of mass from the centre of rotation.

Rotational Inertia and Torque

STOP AND CHECK (PAGE 35)

- Torque is a force applied at a distance from the centre of motion (we'll call it the centre of rotation). More specifically it is the orthogonal/perpendicular/normal (90 degrees) force to the moment arm. You don't have to focus on the last part, it will usually be obvious when a force is a torque. The unit of torque is Nm, not to be confused with Nm^{-1}.

- Rotational inertia is the angular version of mass, the thing that determines how much a body will accelerate when a torque is applied. It can be thought of as how difficult it is to change the speed that something is rotating. The unit is kgm^2.

Angular Momentum

STOP AND CHECK (PAGE 37)

- We have two equivalent ways of calculating the angular momentum of something, we can use the angular version of the momentum equation: $L = I\omega$, or we can use $L = mvr$
- Angular momentum is not conserved when an external torque acts on a system.
- We know that rotational inertia is given by the distribution of mass from the centre of rotation, so when the distribution of mass gets closer to the centre of rotation, the rotational inertia decreases.

Rotational Kinetic Energy

STOP AND CHECK (PAGE 40)

- The formula for rotational kinetic energy is essentially the same as for linear kinetic energy but we replace the linear quantities with angular quantities, so we have

$$E_{k(rot)} = \tfrac{1}{2}(I\omega^2)$$

- In mechanics, the two main types of energy that we deal with are rotational kinetic energy and linear kinetic energy.
- Rotational inertia is a measure of how difficult it is to change the angular speed of an object. Sometimes, this is said to be how hard it is to start or stop something from rotating.

- Yes, it will. Assuming that the same amount of torque is applied to two objects, for the one with lower rotational inertia, the angular acceleration during the time the torque is applied will cause the angular frequency (the number of times it spins) to be larger.

Rotational Motion

QUICK QUESTIONS (PAGE 40)

- Class A's torpedo will recoil faster. Based on Newton's third law, recoil will be induced due to the equal and opposite force that is caused by the torpedo being discharged linearly. Because some of B's torpedo has some of its initial energy being converted to E_kRot, it will have less linear kinetic energy causing the recoil force.
- $I = 0.5mr^2$
  $I = 0.5(1400)(0.2)^2$
  $I = 28kgm^2$
- Torpedos will begin with no angular velocity, therefore:
  - $\omega_i = 0rads^{-1}$
  - $\omega_f = 15rads^{-1}$
  - $\tau = 200\ Nm$
  - $I = 28kgm^2$

$$\tau = I\alpha$$
$$200 = (28)\alpha$$
$$\frac{200}{28} = \alpha \ \ (\alpha \approx 7.14)$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$
$$(15)^2 = (0)^2 + 2(\frac{200}{28})\theta$$
$$\frac{15^2}{2(200/28)} = \theta$$
$$\theta = 15.75rad$$

Simple Harmonic Motion

The Basics

STOP AND CHECK (PAGE 42)

- Criteria for SHM:
    1. An object moves back and forth through some equilibrium position.
    2. There is a ‘restoring force’ that always points in the direction of the equilibrium position. In other words, the restoring force is in the opposite direction to displacement.
    3. The restoring force is proportional to the displacement.
- Here, a ball bouncing elastically between two walls would not be SMH. Firstly the ball would not be moving back and forth through an equilibrium position, as gravity would be pulling the ball down. Secondly, there is no restoring force that acts to bring the ball back into an equilibrium position until the ball hits the wall. Finally, because the restoring force doesn’t occur until it hits a wall, the restoring force is not proportional to displacement.

Terminology

STOP AND CHECK (PAGE 43)

- We call it a restoring force because it always seeks to bring the object back to the equilibrium position, restoring it to its natural position where it would be if no forces were acting on it. It gives us SHM because if there is always a force pointing towards equilibrium (except for when it’s AT equilibrium) then it will continue to move through the equilibrium position.
- Frequency is how many times it oscillates per second; angular frequency is how many times the object moves through 2π radians per second. We can model the movement of anything in SHM as moving around a circle, which is where this measurement comes from. Amplitude is how far away from equilibrium the object travels. A period is how long it takes to complete an oscillation.

Phasor Diagrams

STOP AND CHECK (PAGE 43)

- Acceleration is in the opposite direction to displacement. We can see this because the force experienced is in the opposite direction to displacement, an acceleration is always in the same direction as the force experienced. The direction of velocity is in the direction that displacement is changing, so if displacement is growing, the velocity is in the same direction, if it is shrinking, the velocity is in the same direction.
- These two sets of equations describe the start point that SMH starts from. The first set of equations: $(y = Asin\omega t,\ v = A\omega cos\omega t,\ a =- A\omega^2 sin\omega t)$ are all used whenever we start with t=0 from the equilibrium position. On a phasor diagram this would be when $\theta = 0$ on the right-hand side.
  The second set of equations: $(y = Acos\omega t,\ v = A\omega sin\omega t,\ a =- A\omega^2 cos\omega t)$ will be used whenever we start with t=0 from the maximum position (or when $\theta = \frac{\pi}{2}$).

Mass on a Spring

STOP AND CHECK (PAGE 46)

- Absolutely, this is all the information we need, we will first need to find the mass of the spring by using F = mg and then use: $T = 2\pi\sqrt{\frac{m}{k}}$
- We can change the period by changing mass, spring constant or both. Let's think about just changing one though, given that mass is constant, the period is inversely proportional to √k so we can decrease the spring constant. Given that the spring constant is the same the period is proportional to √m so we can increase the mass and the period will increase.

Pendulum

STOP AND CHECK (PAGE 48)

- It is the force due to gravity that causes a pendulum to undergo SHM. More specifically, it is one of the components of the gravitational force that causes it.

- As the arc gets larger, the force pointing inwards also gets larger, which means that it is proportional to the displacement from equilibrium!
- Tension does not act along the arc, so doesn't contribute to periodic motion.

Energy in SHM Motion

STOP AND CHECK (PAGE 50)

- It doesn't have any potential energy if we ignore the fact that it's probably suspended from the ground, which is reasonable to ignore because the height above the ground makes a very tiny difference to its motion. At equilibrium, the energy of the pendulum is all kinetic baby.
- For an instant at maximum displacement the mass is changing direction, so it doesn't have any kinetic energy; the energy is all elastic potential energy.
- We know that the total energy remains the same, so $E_{total} = E_k + E_p$ so we can rearrange this and get $E_k = E_{total} - E_p$
  $E_k = 3J - 1.1J$
  $E_k = 1.9J$
- We assume that no energy is converted into other forms, so there is no air resistance or internal friction inside the spring. This assumption isn't met in every single real-life system. There is always some amount of friction, air resistance or other influencing effects, which will gradually cause the amplitude to decrease until oscillation stops.

Forced SHM

STOP AND CHECK (PAGE 51)

- The natural frequency is the frequency that something will oscillate at if there is no external intervention, so no damping and no resonance involved, everything has a natural frequency. A good way of thinking about this is when you hit a tuning fork, it will always have the same pitch because it is oscillating at its natural frequency. The natural frequency of something depends on its physical properties.
- We use the equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{5}{20}}$$

$$T = 2\pi\sqrt{0.25}$$

$$T = \pi \text{ seconds} = 3.14\text{s (3s.f.)}$$

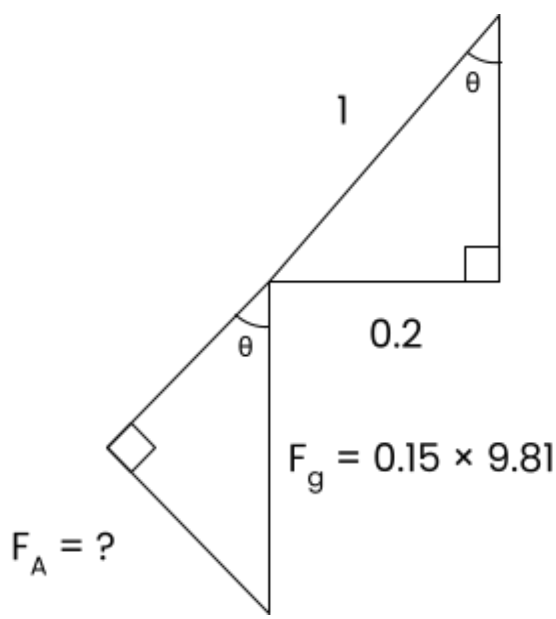
So, the resonant frequency will be $\frac{1}{T}$ = $\frac{1}{\pi}$Hz

When we apply a force near the natural frequency we cause an added acceleration, which causes the mass to overshoot its previous amplitude, increasing the amplitude and the overall energy of the system. The energy is coming from whatever is pushing or pulling the spring, e.g. your arms.

Forced SHM

QUICK QUESTIONS (PAGE 51)

- Simple harmonic motion



- $\sin\theta = \frac{0}{H}$
  - $\theta \sin^{-1}(\frac{0.2}{1})$
  - $\theta = 11.54°$
- $\sin(11.54) = \frac{F_A}{(0.15 \times 9.81)}$
  - $(0.15 \times 9.81)\sin(11.54) = F_A$
  - $F_A = 0.29\text{N}$

