Electrical Systems
ANSWERS

The Basics

The Basics
STOP AND CHECK (PAGE 10)

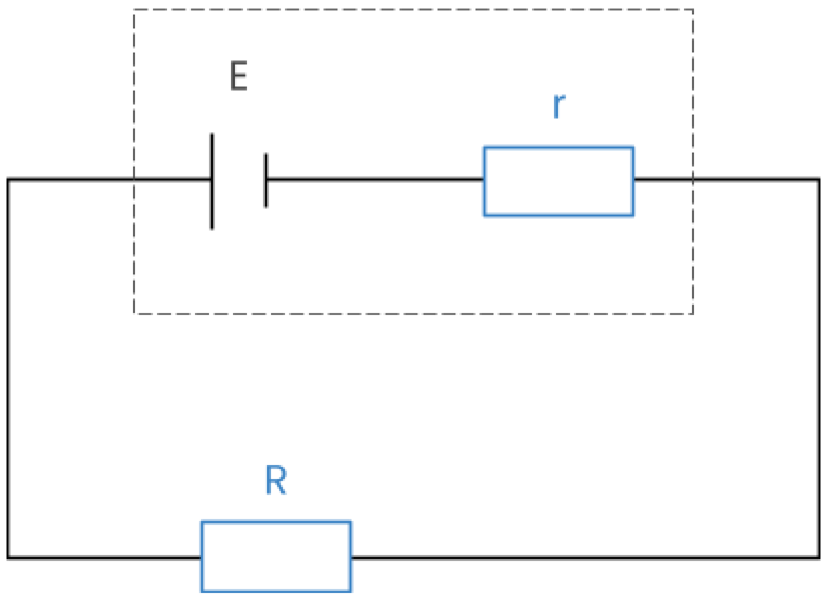
- Current is a measure of how much charge flows past a given point per second. It has the unit Ampere (A) or Cs^{-1}.
- Voltage is the difference in potential electrical energy between two points in a circuit, it has the unit Volt (V) or JC^{-1}.
- Ohm's Law is the relationship between the current and voltage in a circuit. They are related to the resistance of the circuit. Mathematically, this is: $V = IR$
- Power is the amount of energy that a component (or circuit) uses per second. By uses, we mean converts to some other form, such as heat, light or sound.
- You will need to know the current across the branch that has the component and the voltage across the branch (remember that voltage stays the same in parallel and current splits). You can then use Ohm's Law. You can also use the resistance equation for parallel circuits if you have the total resistance and the resistances of the other resistors in the circuit.

DC Electricity

Internal Resistance
STOP AND CHECK (PAGE 11)

- Internal resistance is the amount of resistance that a battery has. This internal resistance is simply from the materials inside the battery. There is always some amount of voltage drop across the internal resistor because there is some opposition to the movement of charge. So, the internal resistance reduces the total amount of voltage a battery can supply.

- The symbol **ε** is the Greek letter epsilon and is an electromotive force. This is essentially the voltage of the battery without any current through it (so there is no voltage drop across the internal resistor). **r** is the internal resistance of the battery. The units for EMF are Volts and the units for internal resistance are Ohms.



Kirchoff's Law

STOP AND CHECK (PAGE 16)

- Kirchoff's Current law is that for any junction, the total amount of current going into the junction is equal to the current leaving the junction. This is a statement of conservation of charge. Given a current entering as I_1 and leaving as I_2 and I_3 we have: $I_1 = I_2 + I_3$
- Kirchoff's Voltage Law states that for any closed loop of a circuit, the total voltage is equal to 0. This is a statement of conservation of energy. Given a closed loop of resistors R_1 and R_2 and a voltage source V, we have: $V - IR_1 - IR_2 = 0$
- When charge passes through a battery, it gains electrical potential energy, so there is a positive voltage across the battery. When charge passes through a

component like a resistor, it loses electrical potential energy in the form of heat, which is a negative voltage across the resistor.

DC Electricity

QUICK QUESTIONS (PAGE 16)

- The voltmeter will read a voltage less than 10V because the internal resistance will use up some of the voltage that the battery is supplying.
- Because there are no other components in the circuit, we can just use Ohm's Law:
  - V = Ir
  - r = $\frac{V}{I}$
  - r = $\frac{10}{1.3}$
  - r = 7.69 Ohms

Capacitors

Capacitance

STOP AND CHECK (PAGE 21)

- A capacitor is an electrical component that is able to store charge. The most basic version of a capacitor is a parallel plate capacitor, which is simply two overlapping metal plates that accumulate negative and positive charges.
- By running a current through them, we deposit electrons on one of the plates. This results in excess negative charge on one plate and excess positive charge on the other. The separation of charge results in an electric field between the plates, which can do work.
- They can store energy as an electric field between their plates. In other words, they allow us to store energy to do work later.
- The current will stop entirely because so much charge has accumulated on the plates that no more electrons can flow.

- We can increase the overlap of the metal plates (A), decrease the distance between the plates (d), and add an insulating dielectric (ε_o), according to the equation: C = $\frac{(A\epsilon_0\epsilon_r)}{d}$
- Dielectrics can really be made of anything. Air is a dielectric because it has some ability to be polarised. However, good dielectrics are insulating materials that contain polar molecules: the better the insulator and greater polarizability, the better the dielectric and the larger the dielectric constant.

Energy

STOP AND CHECK (PAGE 25)

- Refer to the diagram in the guide for the circuit. The voltage across the capacitor over time rises as the charge accumulates on the plates. It will rise to the source voltage given enough time. The voltage across the resistor will go from the source voltage and decrease to 0. Refer to the diagrams in the guide for the charging graphs.
- The time constant is the time it takes for the voltage or current across or through a capacitor to rise or fall by 63%. The time constant depends on the resistance of the circuit and capacitance, both of which it is directly proportional to. It can be calculated by using τ = RC.
- Well, strictly speaking, the capacitor will never finish charging or discharging. The current in the circuit will simply slow to a crawl and it will take an extremely long time before another unit of charge is deposited on or removed from the plate. Practically though, it will be 5 time constants.

Capacitors

QUICK QUESTIONS (PAGE 25)

- Q = CV
    - $Q = 1.2 \times 10^{-6 \times 10}$
    - $Q = 1.2 \times 10^{-5}$C
- τ = RC
    - $\tau = 12 \times 1.2 \times 10^{-6}$
    - $\tau = 1.44 \times 10^{-5}$s

- The charging time could be increased by either increasing the amount of resistance in the circuit or increasing the capacitance of the capacitor. The first can be done by adding more resistors to the circuit in series. The second can be done by either increasing the area of the plates or by decreasing the distance between the capacitor's plates.

Electromagnetic Fields

Electrical Fields

STOP AND CHECK (PAGE 27)

- The right-hand grip rule is a way of figuring out the direction of a magnetic field around a moving charge. Your thumb is in the direction of current and your fingers curl in the direction of the induced magnetic field.
- We just said that charge that is moving has its own magnetic field, and we know that magnetic fields interact with one another, so when a current carrying wire moves through a magnetic field it will experience a force. The force will depend on the length of the wire (how many electrons interact) L, the size of the magnetic field B and the size of the current (the faster electrons move, the greater the magnetic field around them) I. This is given by the equation: $F = BIL$

Faraday's Law and Lenz's Law

STOP AND CHECK (PAGE 30)

- Magnetic flux is the magnetic field strength in an area. It is a measure of how much of a magnetic field passes through a given area. It has the unit Weber (Wb) and is given by the equation: $\phi = BA\cos\theta$. However, because we assume that the magnetic field lines are perpendicular to the surface, this is just $\phi = BA$.
- When there is a change in magnetic flux that a wire is in, we have an induced voltage. If the wire is connected in a closed-loop, we have a current that is

induced as a result. Another way of saying this is that a changing magnetic environment results in an induced voltage. This can be calculated using ε = $\frac{-\Delta\Phi}{\Delta t}$

- The symbol ε is e.m.f., which is not to be confused with the e.m.f. of a battery, although they are related. It is a measure of the induced voltage when charges are in a changing magnetic environment or the current in a wire is changing.
- Faraday's Law states that a changing flux induces a voltage. Lenz's Law states that this induced voltage acts in the opposite direction to the change

Electromagnetic Fields

QUICK QUESTIONS (PAGE 30)

- The magnetic field of the cobalt might induce a voltage and a current in the metal pan (because metal is a great conductor). If Dave touches the metal, he might get a shock.
- -B = 0.6 T
    - t = 1.2 s
    - A = 0.126 m^2
    - ϕ = BA
    - ϕ = 0.6 × 0.126
    - ϕ = 0.0756 Wb
    - ε = $\frac{-\Delta\Phi}{\Delta t}$
    - ε = $\frac{-0.0756}{1.2}$
    - ε = -0.063 V
- The EMF is negative because it is opposing the magnetic flux created by the movement of the cobalt.

Inductors

Mutual Inductance

STOP AND CHECK (PAGE 33)

- A transformer is two inductors that are placed close together. This changes the voltage in one loop to be larger or smaller. These are called step-up and step-down transformers, respectively. This happens because the number of turns in one coil is different to the other.
- Transformers work with alternating current (AC) electricity.
- Transformers change the voltage by having either more or less turns in the secondary coil. If there are more turns in the secondary coil, the voltage induced in that coil (by the changing magnetic field in the primary coil) will be larger. If there are fewer turns, the voltage induced will be less. This is called mutual inductance. The voltage can be calculated by using: $\frac{N_s}{N_p} = \frac{V_s}{V_p}$

  When the secondary coil has more turns, it has a greater inductance. This is because there is more charge in the coil (assuming the density of the wire is the same everywhere). So, the ratio $\frac{N_s}{N_p}$ is greater than 1 and hence the voltage in the secondary coil is greater than that in the first. The power remains the same, so the current in the second coil must drop to compensate.

Self-Inductance

STOP AND CHECK (PAGE 31)

- An inductor is just a coil of wire that opposes a change in current by inducing its own opposing current. This means that when there is a change in current in the circuit, a back EMF will be induced. The inductor is also doing work, so while it is doing this there is energy stored in the magnetic field that is induced.
- If there is a change in current in the circuit, this will generate a magnetic field around the coil. This will induce a back EMF, which creates a current in the opposing direction to the original current.

- We only need to know the current in the circuit and the size of the inductance to calculate how much energy is stored in the magnetic field. This is given by the relationship $E = ½LI^2$. So, half of this energy is dissipated as heat through the resistor/resistance of the circuit, as in RC circuits.

RL Circuits

STOP AND CHECK (PAGE 36)

- An RL circuit is just an inductor (a coil/solenoid) in series with a resistor and some power supply (a battery or AC generator).
- When the switch is closed, the current starts at 0 and increases exponentially until it reaches maximum current. This is because there is a big initial change, which the inductor opposes, but, as the inductor gets used to the current, it opposes the change less until it doesn't have any effect. As such, the resistor's voltage starts at 0 and increases exponentially until it matches the source voltage.

Inductors

QUICK QUESTIONS (PAGE 36)

- The negative sign means that the inductor is producing a back EMF that is opposing the power supply current's direction.
- $-\varepsilon = -L(\frac{\Delta l}{\Delta t})$
  - $L = \frac{-\epsilon \Delta t}{\Delta l}$
  - $= \frac{-(-3.2 \times 12)}{1.7}$
  - = 22.588H
- $E = ½LI^2$
  - $= ½ \times 22.588 \times 1.7^2$
  - = 32.64 J
  - = 33 J (2 sf)

 Level 3 Physics | Electrical Systems |

AC

RMS Voltage and RMS Current

STOP AND CHECK (PAGE 38)

- AC voltage and current can be modelled by sine graphs. They rise and fall with a given frequency and have an amplitude equal to the maximum voltage and current of the circuit. These can be modelled by the following equations:
  - $V = V_{max}\sin(\omega t)$
  - $I = I_{max}\sin(\omega t)$

  Where V and I are the instantaneous voltage and current at a given time.
- Voltage and current are in phase when the only component in the circuit is a resistor. The term **in phase**, means that the voltage in the circuit and the current rise and fall at the same time.
- RMS stands for root mean squared and is the way that we calculate the average of sine waves. We need to use it because the true average of a sine wave is 0, as the current and voltage spend as much time on either side of 0.

AC Capacitors

STOP AND CHECK (PAGE 38)

- Reactance is like resistance and has the same unit Ohms (Ω). We sometimes call this capacitive reactance or X_C. It is inversely proportional to the angular frequency (ω) and capacitance (C). This is given by the equation $X_c = \frac{1}{\omega C}$ Reactance is the opposition to the movement of charge (current), and it is caused by the repulsion of electrons on the plate of a capacitor and charges in the rest of the circuit.
- Supply voltage leads to the capacitor voltage.
- Refer to the diagram in the Guide.

The RLC Circuit

STOP AND CHECK (PAGE 43)

- See the phasor diagram in the section **The RLC Circuit**.
- V = IZ
- The resistor's resistance is still called resistance in an AC circuit. The capacitor and inductor's resistance are called reactance in an AC circuit. The total resistance in an AC circuit is called the impedance.

Resonance

STOP AND CHECK (PAGE 44)

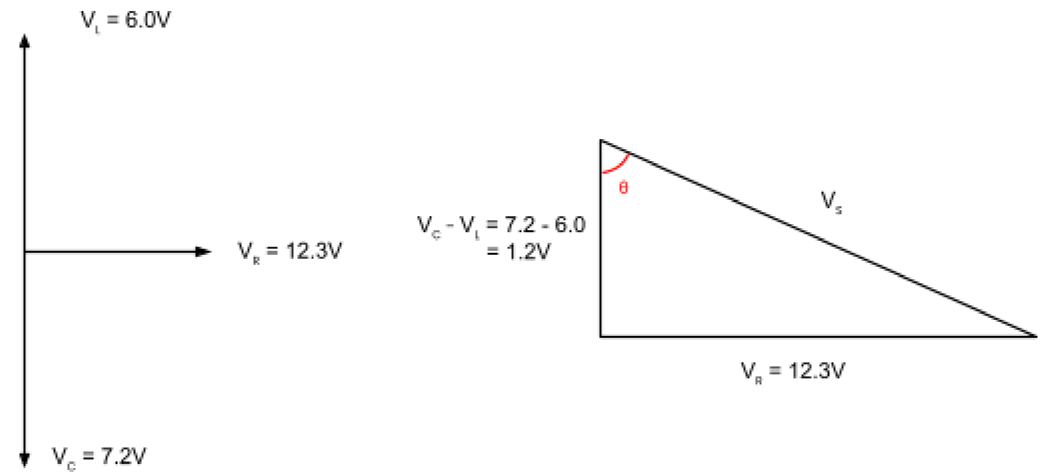
- Reactance is the AC circuit equivalent of resistance for a capacitor and an inductor.
- Impedance is the phase-dependent total resistance and reactance of the circuit. The impedance is given by: $Z = \sqrt{(X_c - X_L)^2 + R^2}$
- A phasor diagram shows the relationship between the voltages, currents and reactances of each component of an AC circuit, it should show the inductor values leading resistor values by 90° and capacitor values lagging by 90°. See The Guide for phaser diagrams.
- X_L leads R by 90° and X_c lags by 90°. This means that X_c and X_L are 180° out of phase or have the opposite phases and so cancel out to give total reactance.
- Resonance is when the reactance of the capacitor and inductor are equal to give the minimum impendence. This means that the impedance is Z = R and that the current and the power in the circuit is at a maximum. The resonant frequency can be found by using the equation: $f_o = \frac{1}{2\pi\sqrt{LC}}$

AC

QUICK QUESTIONS (PAGE 45)

- Using the second diagram and trigonometry, we can find the angle in red:
  - $\tan(\theta) = \frac{O}{A}$
  - $\tan(\theta) = \frac{12.3}{1.2}$
  - $\theta = \tan^{-1}(\frac{12.3}{1.2})$

- $\theta = 84.4^{o}$



- Let's look at $X_L = \omega L$ and $\omega = 2(\pi)f$. If f is decreased, ω also decreases. If ω decreases, then X_L will decrease too. So, if the source frequency is decreased, the reactance of the inductor will also decrease.
- The current can be maximised in an AC circuit by making the reactance of the conductor and the inductor equal. This will minimise impedance, which means that the current will be at a maximum.

