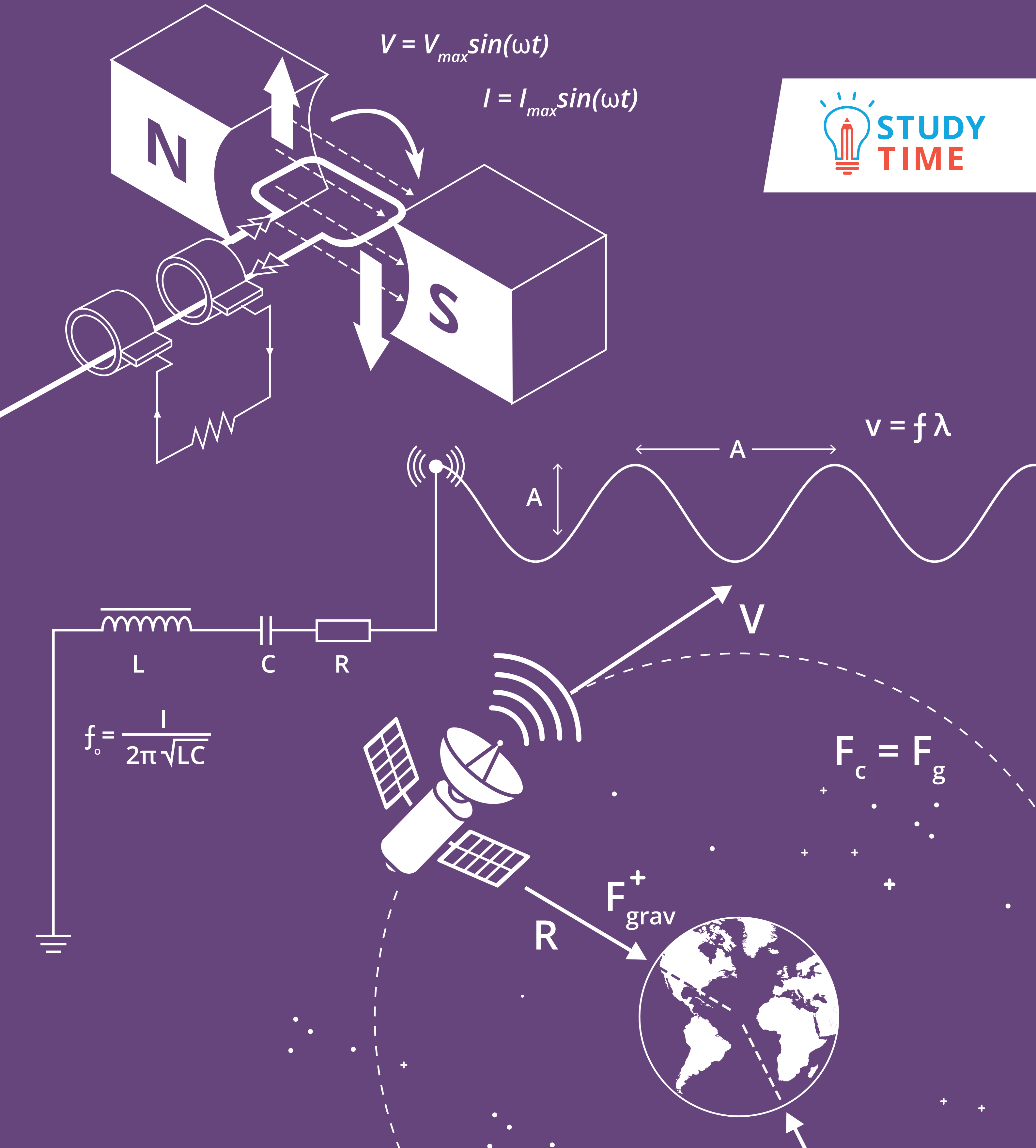



LEVEL 3 PHYSICS

# MECHANICAL SYSTEMS

NCEA Workbook Answers

# 1. Symbols

These are all the formulae you will be given in the exam. Test your knowledge of what each symbol means below. (We have skipped the basic ones that you will have learnt last year).

$$F = ma$$
$$P = mv$$
$$\Delta p = F\Delta t$$
$$\Delta E_p = mg\Delta h$$
$$W = Fd$$
$$E_{K(LIN)} = \frac{1}{2}mv^2$$
$$X_{COM}\ \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$
$$d = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$
$$\omega = \frac{\Delta\theta}{t}$$
$$\alpha = \frac{\Delta\omega}{\Delta t}$$
$$\omega = 2\pi f$$
$$f = \frac{1}{T}$$
$$E_{K(ROT)} = \frac{1}{2}I\omega^2$$
$$\omega_f = \omega_i + at$$
$$\theta = \frac{\omega_f + \omega_i}{2}t$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$
$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$
$$\tau = 1a$$
$$\tau = Fr$$
$$L = mvr$$
$$\theta = \omega_i t - \frac{1}{2}\alpha t^2$$
$$F_g = \frac{GMm}{r^2}$$
$$F_c = \frac{mv^2}{r}$$
$$L = I\omega$$
$$F = -ky$$
$$E_p = \frac{1}{2}ky^2$$
$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$y = A\sin\omega t$$
$$v = A\omega\cos\omega t$$
$$a = -A\omega^2\sin\omega t$$
$$a = -\omega^2 y$$
$$y = A\cos\omega t$$
$$a = -A\omega\sin\omega t$$
$$a = -A\omega^2\cos\omega t$$

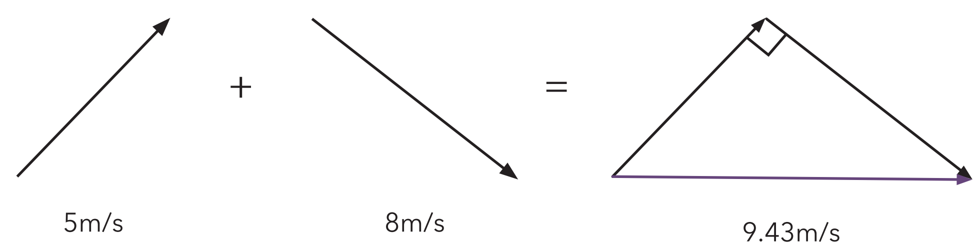
**a.** Fill out the table with the corresponding quantity and unit:

| Symbol | Quantity | Unit |
|---|---|---|
| $x_{com}$ | Distance to the centre of mass | m |
| $v_{com}$ | Velocity of centre of mass | $ms^{-1}$ |
| $p$ | Momentum | $kgms^{-1}$ |
| $\theta$ | Angular position | rad |
| $\omega$ | Angular velocity | $rads^{-1}$ |
| $I$ | Rotational Inertia | $kgm^2$ |
| $\tau$ | Torque | Nm |
| $\alpha$ | Angular acceleration | $rads^{-2}$ |
| $G$ | Universal gravitational constant | $Nm^2kg^{-2}$ |
| $M$ | Mass of larger object | kg |
| $L$ | Angular momentum | $kgm^2s^{-1}$ |
| $y$ | Displacement | m |
| $k$ | Spring constant | $Nm^{-1}$ |
| $A$ | Amplitude | m |
| $T$ | Period | s |

**NOTE**: 'Angular' and 'Rotational' mean the same thing

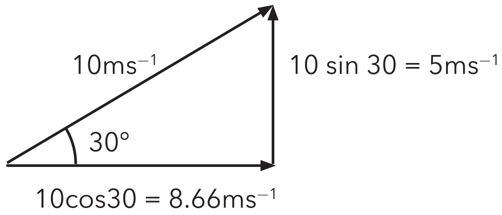
## 2. Vectors

**a.**



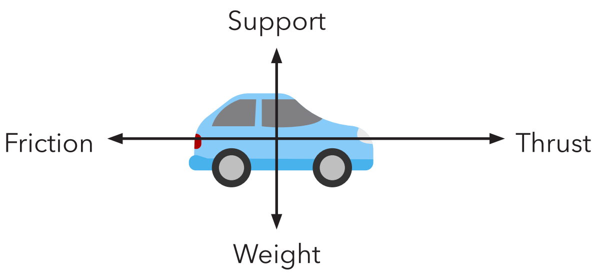
**b.** *i.*




*ii.*


## 3. Free Body Diagrams

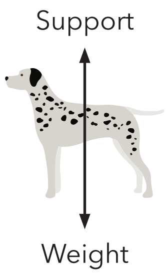
**a.**




*i.* Yes, if the car is accelerating there must be a net force in the forward direction.

*ii.* The thrust and friction force would now be equal and there would be no net force acting.

**b.**




*i.* No, since the forces are equal and opposite.

Part One

# Translational Motion

## 1. Centre of Mass

**a.** The point at which all of the masses in a system are weighted according to their locations and masses to find the average position. It can also be thought of as the balance point.

**b.** The centre of mass accelerates.

**c.** *i.* $x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

Let $x_1$ be the distance from the 2kg mass. Since this is our reference point, $x_1 = 0$ so $m_1x_1 = 0$

$x_{com} = \frac{0 + 8 \times 4}{2 + 8} = 3.2m$ from the 2 kg mass.

*ii.* The centre of mass will move with constant velocity as there are no external forces acting on the system.

*iii.* Before the collision, the centre of mass will move with constant velocity as there are no external forces acting on the system. After the collision, there are still no external forces acting on the system, therefore the centre of mass will still move at a constant velocity.

**d.** *i.* The centre of mass will travel in a parabolic path since the ball is following projectile motion.

*ii.* The centre of mass will remain travelling in the parabolic path since no external forces have been applied to the system. The centre of mass of the system will also remain constant even though the pieces are now distributed in different places.

**e.** *i.* $\frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

*ii.* $x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$7.3 = \frac{(60 \times 0) + 6x + (80 \times 14)}{60 + x + 80}$

$7.3(140 + x) = 0 + 6x + 1120$

$1022 + 7.3x = 6x + 1120$

$1.3x = 98$

$x = 75.4 \text{ kg}$

*iii.* The centre of mass will also move to the left as Bill's mass is now located closer to the left.

**f.** *i.* $v_{com} = \frac{m_1v_1 + m_1v_2}{m_1 + m_2} = \frac{(0.215 \times 5) + (0.130 \times 0)}{0.215 + 0.130} = 3.12ms^{-1}$

Answers

*ii.* An external force has acted on the system (the wall) therefore the centre of mass will accelerate.

## 2. Gravitational Forces

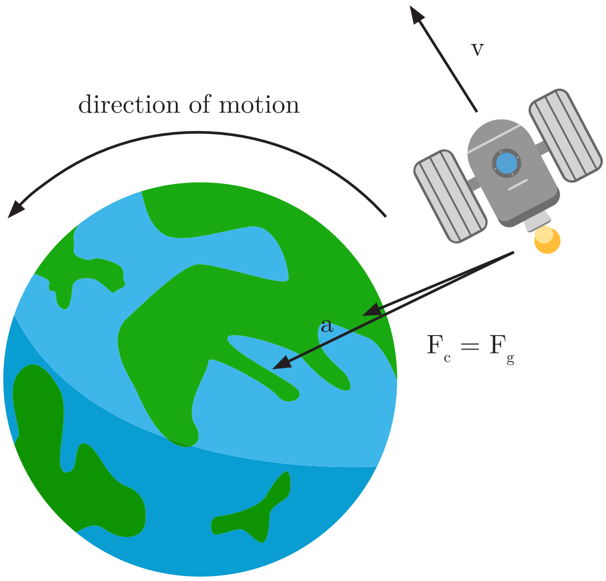
**a.** $F_g = \frac{GMm}{r^2}$

*i.* The universal gravitational constant $(6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2})$. This describes the applied force due to the mass of an object.

*ii.* By increasing the masses of the objects or decreasing the distance between them.

*iii.* For a satellite orbiting the Earth or any object orbiting a planet.

**b.** *i.*




**c.** A satellite with a period of rotation of 24 hours. It must be located directly over the equator and must rotate in the same direction as the Earth. Since the Earth's period of rotation is also 24 hours, the satellite remains at the same point above the Earth at all times.

**d.** The minimum velocity needed to escape from the gravitational influence of a body (a body refers to another object e.g. Earth).

**e.** The velocity at which an object orbits another body (e.g. the velocity at which a satellite orbits Earth).

**f.** For an object to orbit Earth, its centripetal force is provided by the gravitational force, hence, $F_c = F_g$ Therefore:

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

Cancel the $m$ and $r$

$v^2 = \frac{GM}{r}$

$v = \sqrt{\frac{GM}{r}}$

**g.** Escape velocity is always greater than the orbital velocity.

**h.** $F_c = F_g = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 5.972 \times 10^{24} \times 400}{(2{,}000{,}000 + 6{,}371{,}000)^2} = 2273.8N$

**i.** *i.* T = 24 hours = 24 hours × 60 minutes × 60 seconds
= 86400s

*ii.* Gravitational force.

*iii.* Towards the centre of the Earth.

*iv.* $F_g = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 5.972 \times 10^{24} \times 547}{(27.1 \times 10^7)^2} = 2.97N$

*v.* $v = \frac{d}{T} = \frac{2\pi r}{T} = \frac{2\pi \times 27.1 \times 10^7}{86400} = 19708ms^{-1}$

## 3. Momentum

**a.** Impulse is a change in momentum due to a force acting over a certain period of time.

**b.** In elastic collisions, the objects must bounce off each other. Total kinetic energy and momentum are conserved. In inelastic collisions, the objects MAY bounce off each other. Momentum is conserved but total kinetic energy isn't.

**c.** When there are no external forces acting on the system.

**d.** *i.* $p_1 = m_1 v_1$
$p_1 = 2 \times 7$
$p_1 = 14\ kgms^{-1}$

$p^2 = m^2 v^2$
$p_2 = 4 \times 1$
$= 4\ kgms^{-1}$

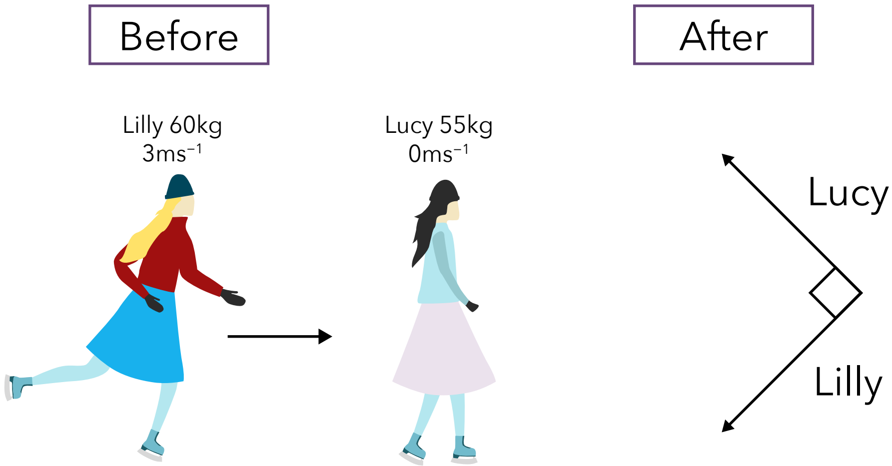
*ii.* Momentum.

*iii.* $p_{before} = p_{after}$
$p_1 + p_2 = (m_1 + m_2)v_{final}$
$14 + 4 = (2 + 4)v_{final}$
$v_{final} = 3ms^{-1}$ to the right

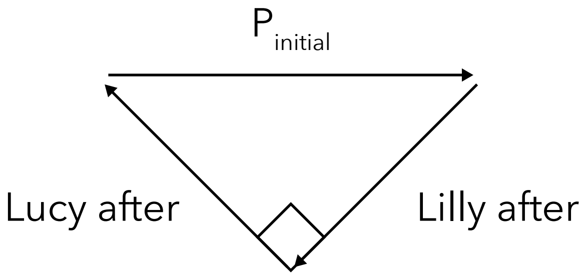
Answers

**e.** *i.*




*ii.*




*iii.* $p_{initial}{}^2 = p_{lucy}{}^2 + p_{lilly}{}^2$

$p_{lucy}{}^2 = p_{initial}{}^2 - p_{lilly}{}^2$

$p_{lucy}{}^2 = (60 \times 3)^2 - (60 \times 1.5)^2$

$p_{lucy} = 156\text{kgms}^{-1}$

$p_{lucy} = mv$

$v = \frac{156}{55}$

$v = 2.84\text{ms}^{-1}$

**f.** *i.* Calculate the force exerted on the cyclist's head.

$\Delta p = Ft$

$F = \frac{2.8}{0.2} = 14\text{N}$

*ii.* The helmet acts as a crumple zone. This is designed to crumple upon impact to absorb the force. The change in momentum (impulse) will remain the same, however, a helmet would increase the time over which the collision takes place. This would reduce the force exerted on the cyclists head as it provides a barrier between the cyclist's head and the ground.

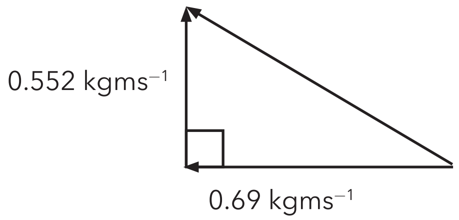
**g.** *i.* $\text{total momentum} = m_1v_1 + m_2v_2$

$= (0.2 \times 3.45) + (0.2 \times 2.76) = 1.24\text{kgms}^{-1}$



*ii.*




*iii.* $p_{final}{}^2 = p_1{}^2 + p_2{}^2$

$p_{final}{}^2 = (0.2 \times 3.45)^2 + (0.2 \times 2.76)^2$

$p_{final}{}^2 = 0.78$

$p_{final} = 0.884 \text{ kgms}^{-1}$

*iv.* $p_{final} = (m_1 m_2)v$

$v = \frac{0.884}{0.4} = 2.21 \text{ ms}^{-1}$

*v.* $\Delta p = p_{final} - p_{initial}$

$= (2.21 \times 0.2) - (3.45 \times 0.2) = 0.248 \text{ kgms}^{-1}$

**h.** *i.* $\Delta p = Ft$

$= 46 \times 0.274$

$= 12.6 \text{ kgms}^{-1}$

*ii.* The gloves provide a barrier between the punching bag and the boxers hand. This provides more cushioning for the punch, hence, increasing the time over which the collision occurs. As impulse stays constant, this reduces the force experienced by the boxer.



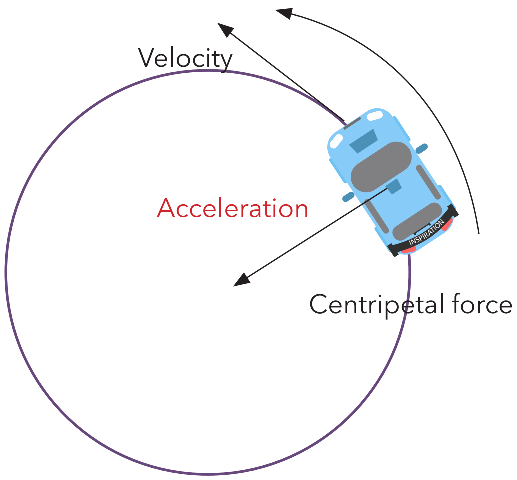
Part Two

# Circular Motion

## 1. Horizontal Circles

**a.** A force that acts on a body moving in a circular motion. It always acts towards the centre of the circle.

**b.** *i.* The angle between the centripetal force vector and the velocity vector is 90 degrees.




*ii.* The friction between the road and the tyres. If this stops acting, the car will travel in a direction tangential to the point at which the force stopped acting.

*iii.* $F_c = \frac{mv^2}{r} = \frac{1000 \times 5^2}{4} = 6250N$

*iv.* $d = 2\pi r$
$= 2 \times \pi \times 4 = 25.1 \text{ m}$

**c.** *i.* The direction of the skater velocity is constantly changing, therefore she is accelerating.

*ii.* $a_c = \frac{v^2}{r} = \frac{2.4^2}{3.5} = 1.65 \text{ ms}^{-2}$

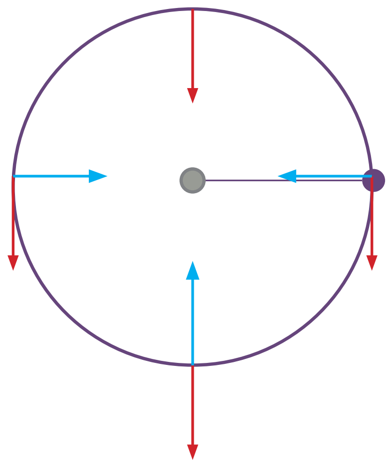
*iii.* Her velocity will still be $2.4\text{ms}^{-1}$, however, now it is directed in a straight line rather than tangential to her motion.

## 2. Vertical Circles

**a.** *i.* Top:
Gravitational/weight force is constant throughout the motion.
Tension force is zero.

Bottom:
Gravitational/weight force is constant throughout the motion.
Tension force is at a maximum and points towards the centre of the circle.

Sides:
Gravitational/weight force is constant throughout the motion.
Tension is directed towards the centre of the circle.



*ii.* Minimum speed – minimum kinetic energy.
Maximum height – maximum gravitational potential energy.

Bottom:
Maximum speed – maximum kinetic energy.
Minimum height – minimum gravitational potential energy.

*iii.* Top: $F_c = F_g$
Sides: $F_c = F_t$
Bottom: $F_c = F_t - F_g$

**b.** *i.* Kinetic and gravitational potential energy.

*ii.* Gravity and the force of the track acting on the car.

*iii.* The friction between the wheels of the car and the track.

*iv.* $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.25 \times 0.53^2 = 0.035J$

*v.* Kinetic and gravitational potential energy.

*vi.* $\frac{1}{2}mv^2 = \frac{1}{2}mv^2 + mgh$

**c.** *i.* $F_c = \frac{mv^2}{r} = \frac{1.3 \times 3.6^2}{0.85} = 19.8N$

*ii.* The weight force only.

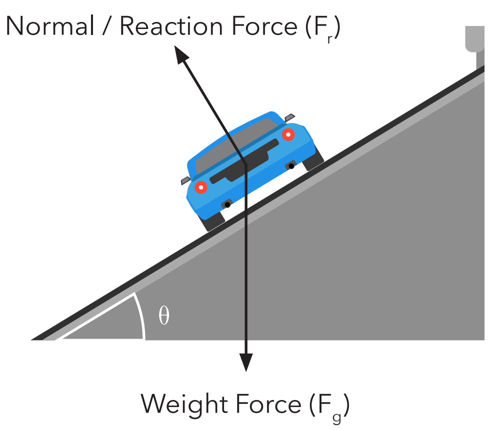
*iii.* The tension force in Mary's arm and the weight force.

*iv.* $\frac{mv^2}{r} = mg$

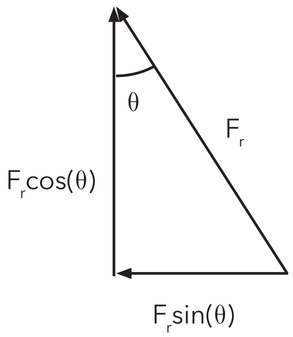
*v.* $\frac{mv^2}{r} = mg$ (Cancel m on both sides)
$v^2 = gr$
$v = \sqrt{gr}$

## 3. Curved Banks

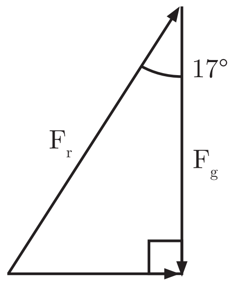
**a.** *i.*




*ii.*



**b.** *i.*




$\cos 17° = \frac{mg}{F_r}$

$F_r = \frac{(3450 \times 9.81)}{\cos 17°}$

$= 35390N$

*ii.* $F_c = F_r \sin(\theta)$

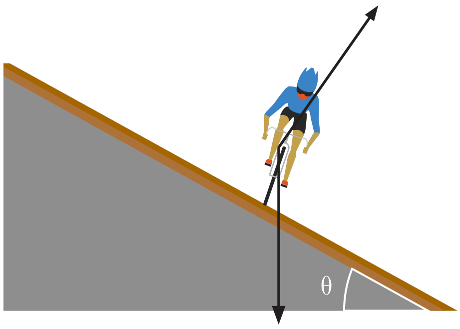
$= 35390 \sin(17°) = 10347N$

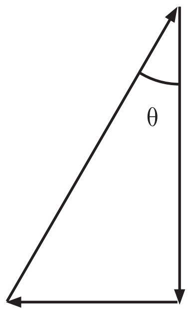
$F_c = \frac{mv^2}{r}$

$v^2 = \frac{F_c \times r}{m} = \frac{10347 \times 4.5}{3450} = 13.5$

$v = 3.67 \text{ ms}^{-1}$

**c.** *i.*




$\cos 10° = \frac{mg}{F_r}$

$F_r = \frac{(93 \times 9.81)}{\cos 10°}$

$= 926 \text{ N}$

*ii.* By splitting the reaction force into its horizontal and vertical components. The horizontal component of the reaction force is the centripetal force.

*iii.* The cyclist needs to adjust their speed to a speed that makes their centripetal force equal to the centripetal force provided by the horizontal component of the reaction force.

Answers

Part Three

# Rotating Systems

## 1. Angular Kinematics

**a.** *i.* $\omega_f = \omega_i + \alpha t$

$= 4 + (1.5 \times 10)$

$= 19 \text{rads}^{-1}$

*ii.* A revolution is rotating an angle of $2\pi$. There are $60$ seconds in a minute.

$19 \text{ rads}^{-1} = \frac{19 \times 60}{2\pi} = 181$ revs per minute.

**b.** *i.* $\omega_f^{\,2} = \omega_i^{\,2} + 2\alpha\theta$

$\theta = \frac{\omega_f^{\,2} - \omega_i^{\,2}}{2\alpha} = \frac{16^2 - 7^2}{2 \times 2} = 51.75 \text{ rad}$

*ii.* $\frac{51.75}{2\pi} = 8.24$ rotations

**c.** $\text{Full rotation} = 2\pi \quad \text{Half rotation} = \pi$

$\theta = \frac{w_f + w_i}{2} t$

$w_f = \frac{2\theta}{t} - w_i = \frac{2\pi}{0.49} - 0.43 = 12.\ 4 \text{ rads}^{-1}$

**d.** A Ferris wheel has an initial rotational velocity of $1.2 \text{ rads}^{-1}$. It accelerates at a rate of $0.012 \text{ rads}^{-2}$ over a time of 50 seconds. Find how many rotations it completes.

$\theta = w_i t + \frac{1}{2}\alpha t^2 = 1.2 \times 50 + (\frac{1}{2} \times 0.012 \times 50^2) = 75 \text{ rad}$

$\text{rotations} = \frac{75}{2\pi} = 11.9 \text{ rotations}$

## 2. Angular Momentum

**a.** Angular momentum is the product of angular velocity and rotational inertia.

**b.** There should be no external torques acting on the system.

**c.** *i.* $L = I\omega = 0.75 \times 2.4 = 1.8 \text{ kgm}^2\text{s}^{-1}$

*ii.* Assuming angular momentum is conserved, the angular velocity would decrease since $L = I\omega$.

Answers

**d.** $L = mvr$

$= 2.5 \times 10^{-6} \times 3.7 \times 10^{9} \times 0.005$

$= 46.25\ kgm^2s^{-1}$

**e.** $L = I\omega$

$\omega = \frac{L}{I} = \frac{2.18}{0.27} = 8.07\ rads^{-1}$

## 3. Rotational Inertia

**a.** A property of matter that opposes rotation. It is a measure of the distribution of mass relative to its centre of rotation.

**b.** $I = mr^2$

**c.** By increasing m or r.

**d.** By decreasing m or r.

**e.** The hollow cylinder, as its mass is located further away from the centre of rotation.

**f.** Bringing his arms in decreases his rotational inertia as his mass is now located closer to his centre of rotation. The formula $L = I\omega$ shows that if rotational inertia decreases, his rotational speed will increase.

**g.** The ballerina should bring her arms as close to her body as possible at the start of the movement, as this will decrease her rotational inertia. She should slowly extend her arms as she wants to slow down, which will increase inertia.

## 4. Torque

**a.** $\tau = Fr$ $\quad\quad \tau = I\alpha$

**b.** *i.* $\tau = Fr = 6 \times 0.4 = 2.4\ Nm$

*ii.* The radius has increased so the force can decrease.

**c.** $I = 2 \times 0.4^2 = 0.32 kgm^2$

$\tau = I\alpha = 0.32 \times 6.4 = 2.048\ Nm$

**d.** $\tau = Fr = 10 \times 0.3 = 3\ Nm$

**e.** *i.* $\tau = \text{I}a$

$\text{I} = \frac{\tau}{a} = \frac{5}{0.46} = 10.87 \text{ kgm}^2$

*ii.* By increasing rotational acceleration or inertia.

## 5. Rotational Kinetic Energy

**a.** A cylinder is rolling down an incline. It has a rotational inertia of $0.6 \text{ kgm}^2$ and an angular velocity of $4.7 \text{ rads}^{-1}$.

*i.* Rotational kinetic energy, linear kinetic energy, gravitational potential energy (there will also be a bit of heat and sound.)

*ii.* $\text{E}_{\text{k(rot)}} = \frac{1}{2} \text{I}\omega^2 = \frac{1}{2} \times 0.6 \times 4.7^2 = 6.63 \text{ J}$

**b.** *i.* $\text{E}_{\text{k(rot)}} = \text{E}_{\text{k(lin)}} = \frac{1}{2} \text{I}\omega^2 = \frac{1}{2} \times 0.58 \times 4.3^2 = 5.36 \text{ J}$

$\text{E}_{\text{k(lin)}} = \frac{1}{2} \text{mv}^2$

$\text{v}^2 = \frac{2\text{E}_{\text{k(lin)}}}{\text{m}} = \frac{2 \times 5.36}{0.5} = 21.44$

$\text{v} = 4.63\text{ms}^{-1}$

*ii.* Rotational kinetic energy is quadrupled since angular velocity is squared in the formula.

**c.** $\text{E}_{\text{k(rot)}} = \frac{1}{2}\text{I}\omega^2$

$\omega = \sqrt{\frac{2\text{E}_{\text{k(rot)}}}{\text{I}}} = \sqrt{\frac{2 \times 64}{0.36}} = 18.9 \text{ rads}^{-1}$

**d.** *i.* Linear and rotational kinetic energy and gravitational potential energy.

*ii.* $\text{E}_{\text{k(total)}} = \text{E}_{\text{k(lin)}} + \text{E}_{\text{k(rot)}} = \frac{1}{2} \text{mv}^2 + \frac{1}{2} \text{I}\omega^2$

$= (\frac{1}{2} \times 0.15 \times 0.75^2) + (\frac{1}{2} \times 0.49 \times 1.7^2)$

$= 0.750 \text{ J}$

## 6. Linear vs. Rotational Motion

**a.** *i.* $\text{d} = \text{r}\theta \qquad \text{v} = \text{r}\omega \qquad \text{a} = \text{r}a$

*ii.* They all have radii in them.

*iii.* In order to convert from rotational to linear, we must multiply by radius.



**b.** *i.* Yes, as no external forces are acting on the system.

*ii.* Both are involved. The ball has linear momentum which is converted to angular momentum when it hits the lever.



Part Four

# Oscillating Systems

**a.** 1. Its restoring force/acceleration is directed towards an equilibrium position.
2. Its restoring force/acceleration is proportional to its displacement away from the equilibrium position.

**b.** 1. Pendulums.
2. Springs.

**c.** The force pulling the object towards the equilibrium position.

**d.** The force is acting in the opposite direction to the displacement.

## 1. Period

**a.** The time that's taken to complete one oscillation.

**b.** $T = \frac{2\pi}{\omega} = \frac{2\pi}{5.2} = 1.21\text{s}$

**c.** $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{4}{9.81}} = 4.01\text{s}$

*i.* By increasing the length of the rope.

**d.** $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{0.82}} = 9.81\ \text{s}$

*i.* By either decreasing mass or increasing spring constant.

**e.** $T = 2\pi\sqrt{\frac{l}{g}}$

$l = (\frac{T}{2\pi})^2 g = (\frac{2.6}{2\pi})^2\ 9.81 = 1.68\ \text{m}$

**f.** $T = 2\pi\sqrt{\frac{m}{k}}$

$m = (\frac{T}{2\pi})^2\ k = (\frac{0.86}{2\pi})^2 \times 2.14 = 0.040\ \text{kg}$

Answers

# 2. Displacement, Velocity and Acceleration in SHM

**a.** $y = A\sin\omega t \qquad v = A\omega\cos\omega t \qquad a = -A\omega^2\sin\omega t$

**b.** $y = A\cos\omega t \qquad v = -A\omega\sin\omega t \qquad a = -A\omega^2\cos\omega t$

**c.** Sine and cosine graphs are periodic and are therefore used to model oscillations.

**d.** *i.* $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.4}{9.81}} = 1.27\text{s}$

Max displacement $t = \frac{1.27}{2} = 0.63\text{s}$

*ii.* $y = A\sin\omega t = 0.25\ \sin\ (3.6 \times 0.47) = 0.248\text{m}.$

**e.** *i.* $v = -A\omega\sin\omega t = -0.04 \times 4.2\ \sin(4.2 \times 0.02) = -0.014\ \text{ms}^{-1}$

*ii.* $a = -A\omega^2\cos\omega t = -0.04 \times 4.2^2\cos(4.2 \times 0.02) = -0.703\ \text{ms}^{-2}$

**f.** *i.* $a = -\ \omega^2 y$

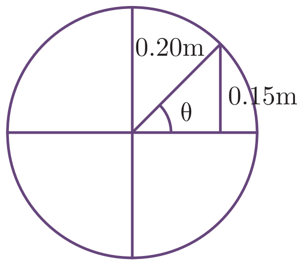
$= -4^2 \times 0.24 = -3.84\ \text{ms}^{-2}$

*ii.* That the acceleration is in the opposite direction to the displacement.

**g.** $y = A\sin\omega t = 0.07\ \sin\ (2.5 \times 0.125) = 0.0215\ \text{m}$

# 3. Reference Circles

**a.** *i.*




$\theta = \sin^{-1}\left(\frac{0.15}{0.20}\right)$

$= 0.848\ \text{rad}$

*ii.* $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.48} = 13.1\ \text{rads}^{-1}$

Answers

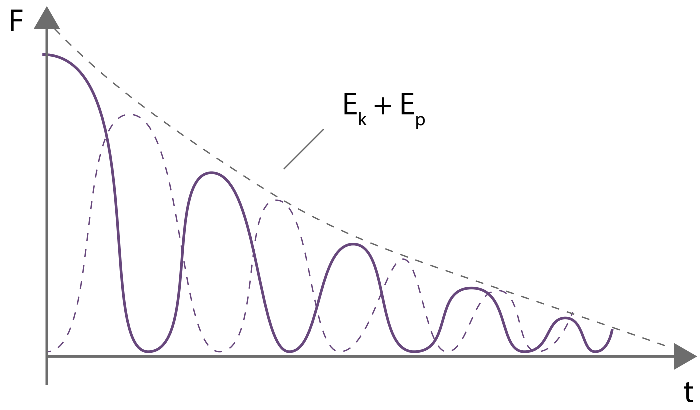
*iii.* $\omega = \frac{\theta}{t}$

$t = \frac{\theta}{\omega} = \frac{0.848}{13.1} = 0.0647 \text{ s}$

## 4. Dampening and Resonance

**a.** When energy in the system is lost to friction, causing the oscillations to become smaller and smaller over time.

*i.*




*ii.* No. This decreases gradually over time.

*iii.* Yes.

**b.** When the frequency of an applied force matches the resonant frequency of the system causing superposition of the oscillations (bigger amplitudes).

*i.* Pushing a child on a swing. You can match the frequency of your pushing to be the same as the natural frequency of the swinging.

**c.** What are the types of energy involved in SHM for pendulums?

Kinetic and gravitational potential energy.

*i.* At the equilibrium position.

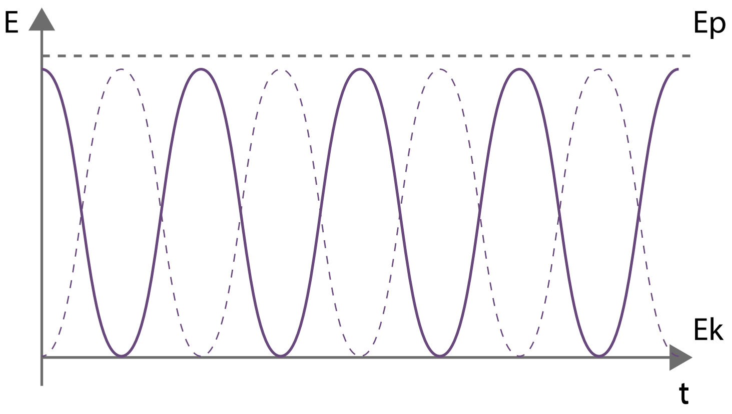
*ii.* At maximum displacement.

*iii.* Yes.



*iv.*




**d.** Kinetic and elastic potential energy.

*i.* At the equilibrium position.

*ii.* At maximum displacement.

## 5. Equations Skills

**a.** $L = mvr$
$L = I\omega$

**b.** $\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$\omega_f = \omega_i + \alpha t$

**c.** $E_{k(lin)} = \frac{1}{2} mv^2$
$E_{k(rot)} = \frac{1}{2} I\omega^2$

**d.** $\tau = I\alpha$
$\tau = Fr$

Answers

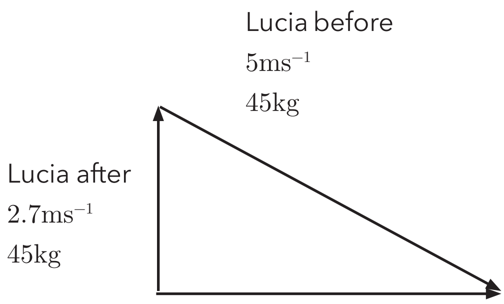
 

Section Two

# Exam Skills & Mixed Practice

## 1. Translational Motion

**a.** *i.*




*ii.* Use your vector diagram to calculate Hannah's final velocity.

$$(p_{lucia\ before})^2 = (p_{lucia\ after})^2 + (p_{hannah})^2$$

$$(p_{hannah})^2 = (p_{lucia\ before})^2 - (p_{lucia\ after})^2$$

$$P_{hannah} = \sqrt{(45 \times 5)^2 - (45 \times 2.7)^2} = 189 \text{ kgms}^{-1}$$

$$v_{hannah} = \frac{P_{hannah}}{m_{hannah}} = \frac{189}{55} = 3.44 \text{ ms}^{-1}$$

*iii.* Reaching her arms out increases the time over which the collision occurs. By considering the impulse formula, $p\Delta = F\Delta t$, we can see that if the time is increased, the force will decrease.

*iv.* $\Delta p = p_{before} - p_{after}$

$= (5 \times 45) - (2.7 \times 45)$

$= 103.5 \text{ kgms}^{-1}$

*v.* $\Delta p = F\Delta t$

$$F = \frac{\Delta p}{\Delta t} = \frac{103.5}{0.68} = 152\text{N}$$

**b.** *i.* $x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(65 \times 0) + (55 \times 5.3)}{65 + 55} = 2.43 \text{ m}$

*ii.* Let the left direction be positive.

$$v_{com} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(65 \times -2.6) + (55 \times 3.9)}{65 + 55} = 0.379\text{ms}^{-1} \text{ to the left}$$

*iii.* $p_{before} = (m_1 + m_2)v_{com}$

$= (55 + 65)0.379 = 45.5 \text{ kgms}^{-1}$

*iv.* This is an elastic collision. Momentum and kinetic energy are conserved.

*v.* Let the left direction be positive.

$p_{before} = p_{after}$

$p_{anna} + p_{mary} = p_{anna} + p_{mary}$

$45.5 = (55v_{anna}) + (65 \times 1.89)$

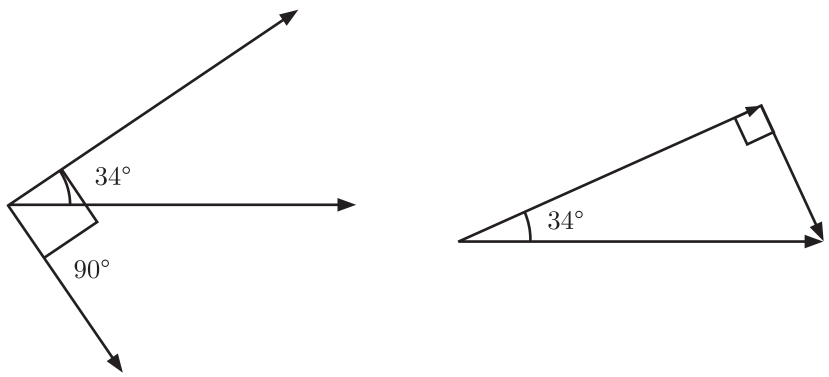
$v_{anna} = -1.41\ ms^{-1}$. The negative sign means she is travelling in the opposite direction.

$v_{anna} = 1.41\ ms^{-1}$ to the right

**c.** *i.* Yes, as there are no external forces acting on the system.

*ii.* No external forces have acted on the system so the centre of mass will continue moving in a parabolic path.

*iii.*




*iv.* $P_{initial}{}^2 = p_1{}^2 + p_2{}^2$

$P_{initial} = \sqrt{(0.36 \times 4.8)^2 + (0.53 \times 3.7)^2}$

$P_{initial} = 2.61\ kgms^{-1}$

$v = \frac{p}{m}$

$v = \frac{2.61}{(0.53 + 0.36)}$

$v = 2.93\ ms^{-1}$

*v.* The helmet has crumpling mechanisms. This means that it will crumple upon impact. The impulse formula shows that $\Delta p = F\Delta t$. The crumpling increases the time over which the impact occurs, hence, reducing the force exerted on the wearer and keeping them safe.



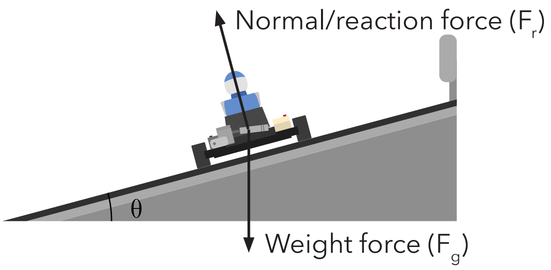
  

## 2. Circular Motion

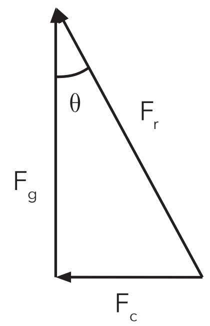
**a.** *i.* Summarise this information into the key points.

- Banked track
- 15°
- 4.5m radius
- 105 kg mass

*ii.*




*iii.*




*iv.* $\tan 15° = \frac{F_c}{F_g}$

$F_c = F_g \tan 15°$

$F_c = mg \tan 15°$

$F_c = (9.81 \times 105) \tan 15°$

$F_c = 276 \text{ N}$

*v.* $F_c = \frac{mv^2}{r}$

$v^2 = \frac{F_c r}{m}$

$v = \sqrt{\frac{F_c r}{m}}$

$v = \sqrt{\frac{276 \times 4.5}{105}} = 3.44 \text{ms}^{-1}$

*vi.* It will stay at a constant height along the track and will not slip up or down.

**b.** *i.* Gravitational potential energy converted to kinetic energy.

*ii.* Kinetic to gravitational potential energy.



*iii.* Weight force only.

*iv.* Weight force, friction force between tyres and track, and normal force.

*v.* Yes. It is provided by a combination of friction and normal force depending on where he is on the loop.

*vi.* For Elliot and his bike to get to the top of the loop de loop, the centripetal force that he experiences needs to be at least as big as the weight force that is acting on him. So,

$F_c = F_g$

$\frac{mv^2}{r} = mg$

$v^2 = gr$

$v = \sqrt{gr} = \sqrt{9.81 \times 3.6} = 5.94ms^{-1}$

*vii.* $\text{Top of slope} = mgh$

$\text{Top of loop} = mg \times 2r + \frac{1}{2}mv^2$

$mgh = mg \times 2r + \frac{1}{2}mv^2$

*viii.* $gh = g \times 2r + \frac{1}{2}v^2$ (divide by m on both sides)

$h = \frac{(2gr + \frac{1}{2}v^2)}{g}$

$h = \frac{(2 \times 9.81 \times 3.6 + \frac{1}{2}5.94^2)}{9.81}$

$h = 9.00 \text{ m}$

**c.** *i.* $F_g = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 89}{(2.4 \times 10^6 + 6.37 \times 10^6)^2} = 462 \text{ N}$

*ii.* The gravitational force would increase.

*iii.* The gravitational force, since it acts towards the centre of Earth.

*iv.* $F_c = \frac{mv^2}{r}$

$v = \sqrt{\frac{F_c r}{m}}$

$v = \sqrt{\frac{462 \times (2.4 \times 10^6 + 6.37 \times 10^6)}{89}} = 6747 \text{ ms}^{-1}$

*v.* $d = 2\pi r$

*vi.* $v = \frac{d}{T}$

$T = \frac{d}{v} = \frac{2\pi r}{v} = \frac{2\pi(2.4 \times 10^6 + 6.37 \times 10^6)}{6747} = 8167 \text{ s}$



vii. $1 \text{ hour} = 60 \text{ seconds} \times 60 \text{ mins}$

$\frac{8167}{(60 \times 60)} = 2.44 \text{ hours}$

## 3. Rotating Systems

**a.** *i.* $\tau = \text{Fr}$

$= 16 \times 0.3$

$= 4.8 \text{ Nm}$

*ii.* Since inertia is proportional to $\text{mr}^2$, the addition of the text book will increase rotational inertia.

*iii.* Assuming that angular momentum is conserved, the increased rotational inertia will decrease the angular velocity due to the relationship $L = I\omega$.

*iv.* $L = \text{mvr} = I\omega$

*v.* $\omega = \frac{\text{mvr}}{I}$

$= \frac{0.3 \times 2.5 \times 0.3}{54}$

$= 4.17 \times 10^{-3} \text{ rads}^{-1}$

**b.** *i.* $\omega_f = \omega_i + \alpha t$

$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{6.3 - 0}{5} = 1.26 \text{rads}^{-2}$

*ii.* Matt, as his mass located further away from his centre of rotation.

*iii.* Gravitational potential energy is converted to linear and rotational kinetic energy.

$E_p = E_{k(rot)} + E_{k(lin)}$

*iv.* Linear kinetic energy.

*v.* Since Matt has a higher rotational inertia, a larger proportion of his kinetic energy will be rotational kinetic energy. Hence, less of his energy will be linear kinetic energy. This means he will reach the bottom after Patrick.

**c.** *i.* $\frac{10.2 \times 60 \text{ seconds}}{2\pi} = 97 \text{ rpm}$

*ii.* $\tau = \text{Fr}$

$\tau = I\alpha$

*iii.* Make the wheel as light as possible and concentrate the mass close to the centre of the wheel.

*iv.* $\theta = \frac{w_f + w_i}{2}t = \frac{14}{2} \times 50 = 350$

Rotations $= \frac{350}{2\pi} = 55.7$ rotations

## 4. Oscillating Systems

**a.** *i.* $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{3}{9.81}} = 3.47\ s$

*ii.* $y = A\cos\omega t$
$v = -A\omega\sin\omega t$
$a = -A\omega^2\cos\omega t$

*iii.* At point of release, $t = 0$
$a = -A\omega^2\cos\omega t$
$a = -2.7 \times 2.4^2\cos(2.4 \times 0)$
$a = -15.6\ ms^{-2}$

*iv.* $y = A\cos\omega t$
$y = 2.7\cos(2.4 \times 1.5)$
$y = -2.42\ m$

*v.* It shows that the pendulum has travelled past the equilibrium position.

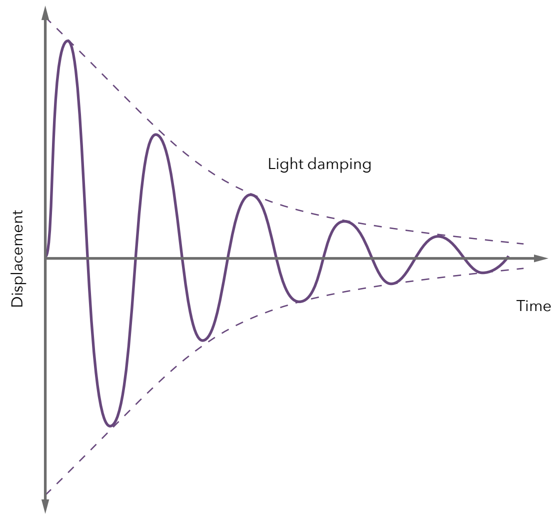
*vi.* $d = y + A$
$d = 2.42 + 2.7 = 5.12\ m$

**b.** *i.* $T = 2\pi\sqrt{\frac{m}{k}}$
$= 2\pi\sqrt{\frac{0.3}{3.73}}$
$= 1.78\ s$

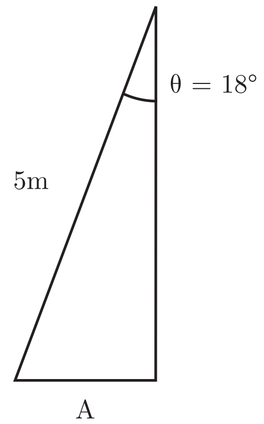
We are looking for the time taken in a quarter of a period, or the time it takes the mass to go from the equilibrium position to maximum displacement.

$T = \frac{1.78}{4} = 0.445s$

*ii.*




**c.** *i.*




$\sin 18° = \frac{A}{5}$

$A = 5\sin 18° = 1.55\text{m}$

*ii.* She has maximum velocity at the equilibrium position. She is momentarily stationary at both end positions.

*iii.* Resonance.

*iv.* Resonance is when the frequency of another source matches the natural frequency of Ruby's swinging. In this case, Brianna is matching the frequency of her pushing to Ruby's swinging.

*v.* Brianna is putting more energy into the system by her pushing. This means Ruby will have a higher amount of kinetic energy that will convert to gravitational potential energy, which means she will swing higher, giving her a greater amplitude.

*vi.* The height of her swing must be kept relatively small otherwise the conditions for SHM will not be met.

Section Three

# Practice Exam

## Question One

**a.** $x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(105 \times 0) + (115 \times 3.8)}{105 + 115} = 1.99 \text{ m}$

(achieved)

**b.** They are considered an isolated system with no external forces acting on them, therefore the velocity of the centre of mass will remain constant as they move closer together.
(achieved)

**c.** $p_{after}{}^2 = p_{buttons}{}^2 + p_{chuckes}{}^2$

$p_{after}{}^2 = (105 \times 5.3)^2 + (115 \times 4.5)^2$

$p_{after}{}^2 = 577498.5$

$P_{after} = 759.9 \text{ kgms}^{-1}$

(achieved)

$v_{after} = \frac{p}{m}$

$v_{after} = \frac{759.9}{(105 + 115)} = 3.45 \text{ ms}^{-1}$

(merit)

**d.** At the top of the circle $F_g = F_c$ (achieved)

$mg = \frac{mv^2}{r}$

$g = \frac{v^2}{r}$

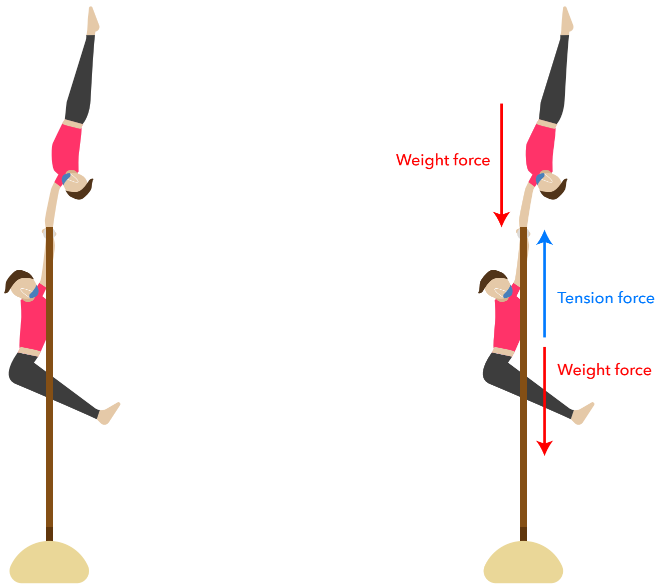
$v = \sqrt{gr} = \sqrt{9.81 \times 0.64} = 2.51 \text{ ms}^{-1}$

(merit)

Answers

**e.** **Note:** this diagram has been changed for clarity in newer versions of the workbook, the new diagram is included below.




At the top of the circle there is no tension force, so the centripetal force is provided by the weight force only ($F_c = F_g$). (merit)

At the bottom of the circle there is both a tension and weight force acting on the acrobat. Since the centripetal force must always act towards the centre of the circle, the tension force must be larger than the weight force ($F_c = F_t - F_g$).
(excellence)

## Question Two

**a.** $\tau = I\alpha = 57 \times 2.45 = 140 \text{ Nm}$

**b.** $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\omega_f^2 = 0 + 2 \times 2.45 \times 2\pi$

$\omega_f^2 = 30.8$

$\omega_f = 5.55 \text{ rads}^{-1}$

(achieved)

$\omega_f = \omega_i + \alpha t$

$t = \frac{\omega_f}{\alpha} = \frac{5.55}{2.45} = 2.27 \text{ s}$

(merit)

**c.** $E_{k(rot)} = \frac{1}{2} I\omega^2$

$\omega = \sqrt{\frac{2E_k}{I}} = \sqrt{\frac{2 \times 345}{57}} = 3.48 \text{ rads}^{-1}$

(achieved)

*i.* Find the linear velocity of the skater if their centre of mass has a radius of $30\ \text{cm}$ from their centre of rotation.

$v = r\omega$

$v = 0.3 \times 3.48 = 1.044\ \text{ms}^{-1}$.

(merit)

**d.**
- Extending their arms and leg out increases their rotational inertia as their mass is now located further away from their centre of rotation. (achieved)
- Their angular momentum will be conserved. Due to the relationship $L = I\omega$, this means her rotational velocity will halve. (both points = merit)
- Rotational kinetic energy is determined by $E_{k(rot)} = \frac{1}{2}I\omega^2$. If inertia has doubled and rotational velocity has halved, due to the $\omega^2$ in the equation, the rotational kinetic energy will halve. (all three points = excellence)

## Question Three

**a.** $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{5}{9.81}} = 4.49\ \text{s}$

$\frac{4.49}{2} = 2.245\text{s}$

(achieved)

**b.** $y = A\sin\omega t \qquad \omega = \frac{2\pi}{T}$

(achieved)

$y = A\sin\left(\frac{2\pi t}{T}\right)$

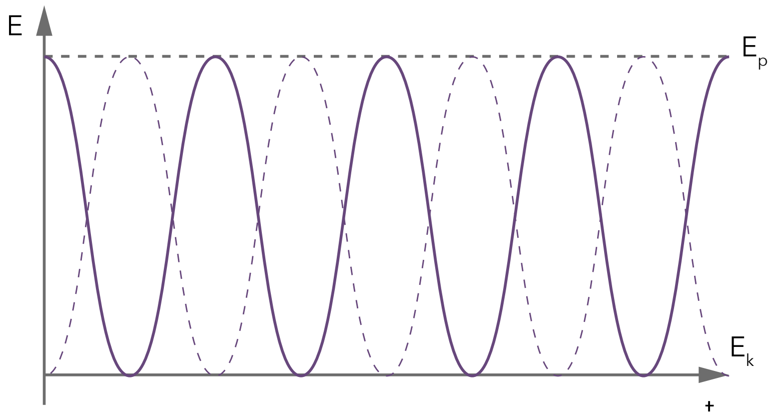
$\sin^{-1}\left(\frac{y}{A}\right) = \frac{2\pi t}{T}$

$t = \sin^{-1}\left(\frac{y}{A}\right) \times \frac{T}{2\pi} = \sin^{-1}\left(\frac{0.7}{1.5}\right) \times \frac{4.49}{2\pi}$

$t = 0.347\ \text{s}$

(merit)

**c.**



At maximum displacement, the acrobat has maximum gravitational potential energy and zero kinetic energy. This converts to kinetic energy as they approach the equilibrium position. At the equilibrium position, they have maximum kinetic energy and zero gravitational potential energy. (merit)

**d.** *i.* The other performer has matched the frequency of the pushing to the natural frequency of the acrobat's swinging. This results in resonance, meaning the amplitude of oscillation increases. (merit)

*ii.* In order for something to be considered SHM, the acceleration must be directed towards the equilibrium position and it must be proportional to the displacement from the equilibrium position. When oscillations become too large, these conditions cannot be met, so it will no longer be considered SHM. (both points for excellence)

