## ? Geometric Reasoning ANSWERS

## Angles and Parallel Lines

## Solving an Angle Problem

## stop and check (page 9)

- An angle describes the space between two intersecting lines at, or near the point that they meet.
- Angles on a straight line add to $180^{\circ}$, angles around a point add to $360^{\circ}$, and vertically opposite angles are equal.
- We use geometric text language to quickly identify angles and sides we're trying to find.


## Solving Parallel Line Problems

stop and check (page וו)

- Alternate angles are equal, corresponding angles are equal, and co-interior angles add to $180^{\circ}$.


## Angles on Parallel Lines

QUICK QUESTIONS (PAGE וו)

- First off, we can see that $\angle \mathrm{p}=180^{\circ}-90^{\circ}-4^{\circ}$ ( $\angle$ 's on a straight line add to $180^{\circ}$ ), so $\angle p=76^{\circ}$.
- Then we can use the fact that corresponding angles are equal to see the angle CGH (which is just the angle between the lines CG and GH) is the same as the angle $p$, which is $86^{\circ}$. We then know that since co-interior angles add to 180:

$$
\angle q=180^{\circ}-\angle p \angle q=180^{\circ}-86^{\circ} \angle q=94^{\circ}
$$

## Circle and Polygon Properties

## Solving a Circle Problem

stop and check (page 17)

- The angle where the radius meets the tangent is $90^{\circ}$.
- Angles at the centre are double the opposite angle on the circumference.
- Angles on the same arc are equal.
- A line that bisects a chord will be $90^{\circ}$ to that chord.
- A triangle in a semi-circle will always be a right-angled triangle.
- The opposite angles in a cyclic quadrilateral are equal.


## Properties of Polygons

STOP AND CHECK (PAGE 19)

- They add to $360^{\circ}$
- You could just use the 'angles on a straight line add to $180^{\circ}$ ' rule with the exterior angles to find the interior angles. Alternatively, use the formula for the interior angles of a regular polygon:

$$
\text { Sum int. } \angle ' s=180^{\circ} \times(\text { no.sides }-2)
$$

Then, divide by the number of corners to get the interior angles.

## Onwards to Triangles

## stop and check (page 20)

- The angles in a triangle add to $180^{\circ}$.
- The base angles of an isosceles triangle are equal.
- The exterior angle of a triangle is the sum of the two opposite interior angles.


## Similar Triangles

STOP AND CHECK (PAGE 23)

- Similar triangles have the same interior angles.
- You can use the scale factor to find the lengths of unknown sides, since each side is just scaled up or down by the scale factor when going from one triangle to the other.


## Circle and Polygon Properties

## QUICK QUESTIONS (PAGE 23)

- The line in the middle is the diameter of the circle, so using our circle properties we can instantly see that:
- $\angle \mathrm{z}=90^{\circ}$ (Angles that bisect the diameter are $90^{\circ}$ )
- $\angle x=90^{\circ}$ (Angles at the centre are double the opposite angle at circumference)
- $\angle y=90^{\circ}$ (Angles at the centre are double the opposite angle at circumference)
- For this one, we'll first need to find the interior angles of this regular pentagon.

So, that's:

$$
180^{\circ} \times(5-2)=540^{\circ}
$$

With 5 corners that makes every interior angle:

$$
\frac{540^{\circ}}{5}=108^{\circ}
$$

So, using the fact that angles on a straight line add to $180^{\circ}$ :

$$
\begin{gathered}
\angle x=180^{\circ}-108^{\circ} \\
\angle x=72^{\circ}
\end{gathered}
$$

- The angles $y$ and $z$ can be found straight away by using a few of our rules:
- $\angle \mathrm{z}=180^{\circ}-37^{\circ}=143^{\circ}$ (angles on a straight line add to $180^{\circ}$ )
- $\angle y=37^{\circ}$ (base angles of isosceles triangles are equal)
- $\angle x=180^{\circ}-37^{\circ}-37^{\circ}=106^{\circ}$ (interior angles of a triangle add to $180^{\circ}$ )


## Pythagoras' Theorem, Trigonometry, and Bearings

## Pythagoras' Theorem

STOP AND CHECK (PAGE 25)

- The components of an exponential graph are:

$$
\begin{gathered}
a^{2}+b^{2}=c^{2} \\
b^{2}=c^{2}-a^{2} \\
b=\sqrt{ }\left(c^{2}-a^{2}\right)
\end{gathered}
$$

## Trigonometry

stop and check (page 28)

- They stand for the formulas:
- Sine $=\frac{\text { Opposite }}{\text { Hypotenuse }}$
- Cosine $=\frac{\text { Adjacent }}{\text { Hypotenuse }}$
- Tangent $=\frac{\text { opposite }}{\text { Adjacent }}$ Opposite/Adjacent
- For cos, press shift > cos to get $\cos ^{-1}$, for sin press shift > sin to get $\sin ^{-1}$, and for tan press shift > tan for $\tan ^{-1}$.


## Bearings

## STOP AND CHECK (PAGE 29)

- They're measured from north in a clockwise direction.
- You draw a north arrow then draw your angle from that. The bearing is the angle from north to whatever direction you're going, so it has to come from north.


## Pythagoras' Theorem, Trigonometry, and Bearings <br> stop and check (page 30)

- For this question, we have to make an addition to our diagram to see how to do the question, by extending the line $A B$ a little bit.
- We know that the angle CBD is $60^{\circ}$ since corresponding angles are equal. We also know that angles on a straight line add to $180^{\circ}$, so the angle from A to D is $180^{\circ}$. We now have everything we need to know, since that $60^{\circ}$ we found, that $180^{\circ}$ we found and the angle ABC all add to $180^{\circ}$. So:

$$
\begin{gathered}
280=\angle A B C+180^{\circ}+60^{\circ} \\
\angle A B C=280^{\circ}-180^{\circ}-60^{\circ} \\
\angle A B C=40^{\circ}
\end{gathered}
$$

## Proofs and Generalised Problems

## How to Answer Proof Problems

stop and Check (page 32)

- The first tip is to add to the diagram, it's a great way to get started on a question and it'll probably give you ideas as you go. The second tip is that isosceles triangles are your best friends. They're really useful for proof questions since the base angles are the same and two of the sides are the same. It gives you a lot of information to work with.


## How to Answer Generalised Problems <br> STOP AND CHECK (PAGE 33)

- You should look for isosceles triangles and parallel lines.


## Proofs and Generalised Problems

## QUICK QUESTIONS (PAGE 34)

- $\angle a=180-\angle b$

