



# NCEA Workbook Answers

# The Foundations

## 1. Scientific Notation

- |           |                            |                            |
|-----------|----------------------------|----------------------------|
| <b>a.</b> | i. $1.00 \times 10^3$      | ii. $1.00 \times 10^5$     |
|           | iii. $3.00 \times 10^6$    | iv. $5.40 \times 10^4$     |
|           | v. $8.25 \times 10^6$      | vi. $1.93 \times 10^6$     |
| <b>b.</b> | i. 100                     | ii. 3,100                  |
|           | iii. 1,000,000             | iv. 257,000                |
|           | v. 98,400,000              | vi. 128,590                |
| <b>c.</b> | i. $1.00 \times 10^{-3}$   | ii. $1.00 \times 10^{-6}$  |
|           | iii. $2.00 \times 10^{-7}$ | iv. $2.40 \times 10^{-3}$  |
|           | v. $7.15 \times 10^4$      | vi. $6.44 \times 10^{-6}$  |
| <b>d.</b> | i. 0.1                     | ii. 0.003                  |
|           | iii. 0.000007              | iv. 0.064                  |
|           | v. 0.000908                | vi. 0.000051186            |
| <b>e.</b> | i. $1 \times 10^{-3}L$     | ii. $4.1 \times 10^5g$     |
|           | iii. $9.8 \times 10^6m$    | iv. $4.5 \times 10^4mL$    |
|           | v. $1.22 \times 10^{-2}g$  | vi. $2.8 \times 10^{-4}km$ |

f. i. 10kg

ii. 3.62m

iii. 7,153L

iv. 6,300km

v. 1,060mg

vi. 1,280,700,000mL

## 2. Naming Variables, Symbols, and Units

a.

Symbol	Variable	SI Unit
d	Distance	Metres, m
t	Time	Seconds, s
v	Velocity	Metres per second, $\text{ms}^{-1}$
$v_i$	Initial velocity	Metres per second, $\text{ms}^{-1}$
$v_f$	Final velocity	Metres per second, $\text{ms}^{-1}$
a	Acceleration	Metres per second per second, $\text{ms}^{-2}$
m	Mass	Kilograms, kg
F	Force	Newtons, N
p	Momentum	Kilogram metres per second, $\text{kgms}^{-1}$
$\Delta p$	Impulse	Kilogram metres per second, $\text{kgms}^{-1}$
$a_c$	Centripetal acceleration	Metres per second per second, $\text{ms}^{-2}$
$F_c$	Centripetal force	Newtons, N
$\tau$	Torque	Newton metres, Nm
W	Work	Joules, J
P	Power	Watts, W
$E_k$	Kinetic energy	Joules, J
$E_p$	Potential energy	Joules, J
x	Spring extension	Metres
k	Spring constant	Newtons per metre, $\text{Nm}^{-1}$
$\Delta$	Change in	(none)

- b. The force acting on an object in order for it to travel on a circular path at a constant speed.
- c. Work is a transfer or conversion of energy. For example, lifting a ball converts your chemical potential energy (in your cells) into gravitational potential energy in the ball, then dropping the ball converts that gravitational potential energy into kinetic energy. Work is done in both cases.
- d. Gravitational potential energy, elastic potential energy, chemical potential energy, nuclear potential energy.
- e. i. Since  $mv$  has units of  $\text{kgms}^{-1}$ , momentum has units of  $\text{kgms}^{-1}$
- ii. Since  $Fd$  has units of  $N \cdot m$ , torque has units of  $Nm$
- iii. Since  $mv^2$  has units of  $\text{kg} \times \text{ms}^{-1} \times \text{ms}^{-1}$ , energy has units of  $\text{kgm}^2\text{s}^{-2}$ , which is another way to write  $J$ . Note that coefficients like  $\frac{1}{2}$  don't change the units.
- f. i. For the units to match, we must have  $\text{ms}^{-2} = \frac{\text{ms}^{-1}}{\text{s}}$ , so the formula must be  $a = \frac{v}{t}$
- ii. For the units to match, we must have  $\text{kgm}^2\text{s}^{-2} = \text{kgms}^{-2} \times m$ , so the formula must be  $W = Fd$
- iii. We notice that seconds only appear in acceleration and velocity. For the seconds to match up, we must have to use the square of velocity. Putting this together, for the units to match, we must have  $\text{ms}^{-2} = \frac{\text{ms}^{-1} \times \text{ms}^{-1}}{\text{m}}$ , so the formula must be  $a_c = \frac{v^2}{r}$

### 3. Pythagoras and Trigonometry

a. i.  $? = \sqrt{1^2 + 2^2}$   
 $? = \sqrt{5}$   
 $? = 2.24$

ii.  $? = \sqrt{3^2 + 4^2}$   
 $? = \sqrt{25}$   
 $? = 5$

iii.  $? = \sqrt{5^2 - 3^2}$   
 $? = \sqrt{16}$   
 $? = 4$

iv.  $? = \sqrt{2^2 - 1^2}$   
 $? = \sqrt{1}$   
 $? = 1$

b. i.  $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$   
 $\tan \theta = \frac{1}{2}$   
 $\theta = \tan^{-1}(\frac{1}{2})$   
 $\theta = 26.57^\circ$

ii.  $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$   
 $\sin \theta = \frac{3}{5}$   
 $\theta = \sin^{-1}(\frac{3}{5})$   
 $\theta = 36.87^\circ$

iii.  $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$   
 $\cos \theta = \frac{4}{5}$   
 $\theta = \cos^{-1}(\frac{4}{5})$   
 $\theta = 36.87^\circ$  (3 s.f.)

iv.  $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$   
 $\cos \theta = \frac{1}{\sqrt{2}}$   
 $\theta = \cos^{-1}(\frac{1}{\sqrt{2}})$   
 $\theta = 45^\circ$

c. i.  $\cos 30 = \frac{1}{x}$   
 $x = \frac{1}{\cos 30}$   
 $x = 1.15$  (3 s.f.)

ii.  $\cos \theta = \frac{1}{2}$   
 $\theta = \cos^{-1}(\frac{1}{2})$   
 $\theta = 60^\circ$

$\theta = 180 - 90 - 30$   
 $\theta = 60^\circ$

$x = \sqrt{2^2 - 1^2}$   
 $x = \sqrt{3}$   
 $x = 1.73$

## 4. Vectors

- a. Vector quantities have direction and magnitude, scalar quantities only have magnitude
- b. The length represents the magnitude and the direction represents the overall direction of the quantity.

c. i. Vector

ii. Scalar

iii. Scalar

iv. Vector

v. Vector

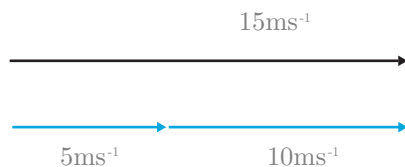
vi. Vector

vii. Scalar

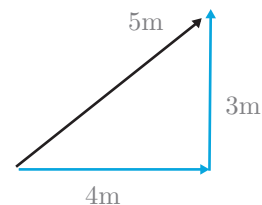
viii. Scalar

ix. Vector

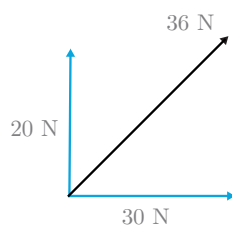
d. i. Magnitude =  $5 + 10 = 15\text{ms}^{-1}$



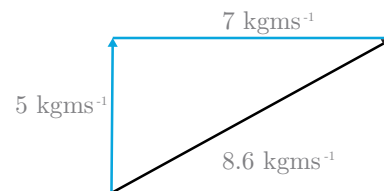
ii. Magnitude =  $\sqrt{4^2 + 3^2} = 5\text{m}$



iii. Magnitude =  $\sqrt{30^2 + 20^2} = 36.06\text{N}$



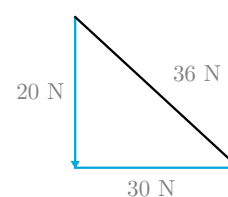
iv. Magnitude =  $\sqrt{7^2 + 5^2} = 8.60\text{kgms}^{-1}$



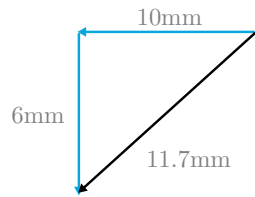
v. Magnitude =  $9 - 6 = 3\text{ms}^{-2}$



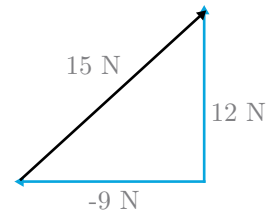
vi. Magnitude =  $\sqrt{30^2 + 20^2} = 36.06\text{N}$



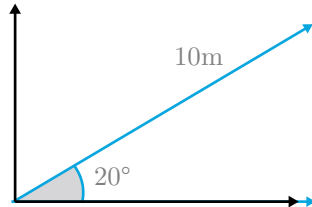
vii. Magnitude =  $\sqrt{10^2 + 6^2} = 11.66\text{mm}$



viii. Magnitude =  $\sqrt{12^2 + 9^2} = 15\text{N}$



e. i.



$$\sin(20) = \frac{\text{vertical}}{10}$$

$$\text{vertical} = 10\sin(20)$$

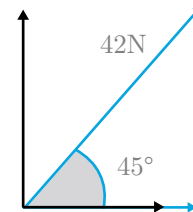
$$\text{vertical} = 3.42\text{m}$$

$$\cos(20) = \frac{\text{horizontal}}{10}$$

$$\text{horizontal} = 10\cos(20)$$

$$\text{horizontal} = 9.40\text{m}$$

ii.



$$\sin(45) = \frac{\text{vertical}}{42}$$

$$\text{vertical} = 42\sin(45)$$

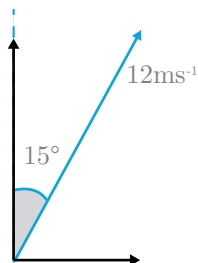
$$\text{vertical} = 29.70\text{N}$$

$$\cos(45) = \frac{\text{horizontal}}{42}$$

$$\text{horizontal} = 42\cos(45)$$

$$\text{horizontal} = 29.70\text{N}$$

iii.



$$\sin(15) = \frac{\text{horizontal}}{12}$$

$$\text{horizontal} = 12\sin(15)$$

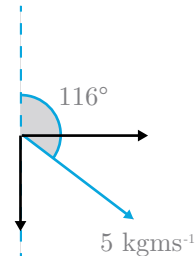
$$\text{horizontal} = 3.11\text{ms}^{-1}$$

$$\cos(15) = \frac{\text{vertical}}{12}$$

$$\text{vertical} = 12\cos(15)$$

$$\text{vertical} = 11.59\text{ms}^{-1}$$

iv.



$$\sin(180-116) = \frac{\text{horizontal}}{5}$$

$$\text{horizontal} = 5\sin(64)$$

$$\text{horizontal} = 4.49\text{kgms}^{-1}$$

$$\cos(64) = \frac{\text{vertical}}{5}$$

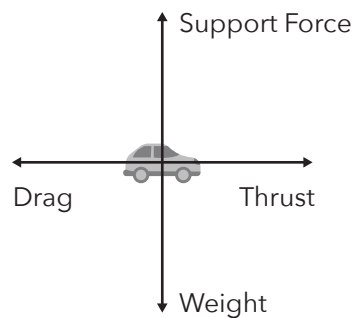
$$\text{vertical} = 5\cos(64)$$

$$\text{vertical} = 2.19\text{kgms}^{-1}$$

## 5. Forces

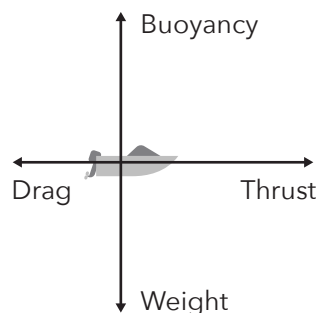
- a. Any object will remain at rest or have a constant velocity unless acted upon by an unbalanced force. That means that if an object has no net acceleration, its net force is  $F = ma = m \cdot 0 = 0$ . So, an object with no net acceleration has no net force and will remain at rest or maintain a constant velocity, according to Newton's first law.
- b. Newton's second law of motion states that the force on an object with a constant mass is  $F = ma$ . So, when a ball is dropped on earth, it will feel an acceleration downwards equal to  $g$ , so the force on the ball will be  $F = ma = mg$ .
- c. For every action (force) there is an equal and opposite reaction. This means that for a ball which is stationary and on the ground, it will exert a force onto the ground since the ball itself is experiencing a weight force. The ground will then give an equal and opposite force (the support force) according to Newton's third law.
- d. Mass is the amount of matter in an object, while weight is the force due to gravity acting on a mass. Mass is measured in kilograms while weight is measured in Newtons.

e.



- i. They're the same size, otherwise, the car would be floating or sinking.
- ii. There can't be a net force in the horizontal direction, otherwise, the car wouldn't be travelling at a constant speed.

f.

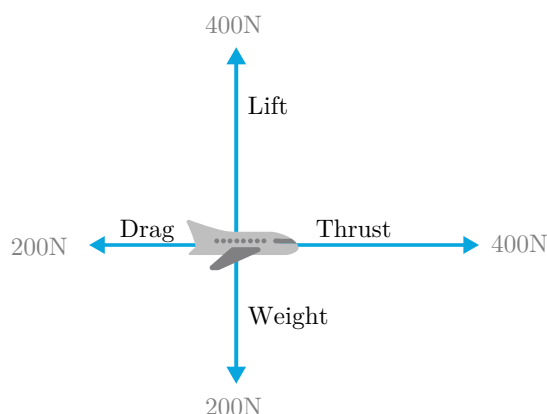


- i. The weight force is still the same size as the buoyancy force, otherwise it would sink in the water or float into the air.
- ii. There has to be a net force to the right so that the boat speeds up.



- iii. For the boat to speed up, it must have a net force in the direction it is moving. If we assume the boat is moving to the right, then it must have a net force to the right. This means that the thrust is bigger than the drag, which creates the net force.

g.



- i. The plane is not in equilibrium and the net force is pointing diagonally to the right and up.
- ii. The net force in the horizontal direction is 200N to the right, and the net force in the vertical direction is 200N upwards.
- iii. The total net force on the plane is  $\sqrt{200^2 + 200^2} = \sqrt{80000} = 282.84\text{N}$  and this force makes an angle of  $45^\circ$  with the horizontal.
- iv.  $F = ma$   
 $a = \frac{F}{m} = \frac{282.84}{20.4} = 13.86\text{ms}^{-2}$
- h. i. For an object to be in equilibrium it must have no net force acting on it. In other words, the forces acting on it add to zero.
- ii. The velocity stays constant. Since there is no net force, there is no net acceleration, which means the velocity isn't changing.

## 6. Kinematics

a. i.  $2\text{km} + 3\text{km} = 5\text{km}$

ii.  $d'^2 = 2^2 + 3^2$   
 $d' = \sqrt{4 + 9}$   
 $= 3.61\text{m}$

iii.  $d = \frac{v_i + v_f}{2} t$   
 $= \frac{0 + 1.80}{2} \times 12$   
 $= 10.8\text{m}$

iv.  $v_f = v_i + at$   
 $= 2.0 + 0.06 \times 25$   
 $= 3.5\text{ms}^{-1}$

v.  $v_f^2 = v_i^2 + 2ad$   
 $v_i = \sqrt{v_f^2 - 2ad}$   
 $v_i = \sqrt{0 - 2 \times -0.26 \times 5.5}$   
 $v_i = 1.69\text{ms}^{-1}$

b. i. The car's speed is changing at a negative rate, which means it is slowing down.

ii.  $v_f = v_i + at$   
 $= 14.7 - 0.18 \times 60$   
 $= 3.9\text{ms}^{-1}$

iii.  $v_f^2 = v_i^2 + 2ad$   
 $d = \frac{v_f^2 - v_i^2}{2a}$   
 $d = \frac{0 - 3.9^2}{2 \times -0.18}$   
 $d = 42.25\text{m}$

c. i. Just before it hits the ground

ii.  $v_f = v_i + at$   
 $= 0 + 9.8 \times 2.50$   
 $= 24.5\text{ms}^{-1}$

iii.  $d = v_i t + \frac{1}{2} at^2$   
 $d = (0 \times 1) + (\frac{1}{2} \times 9.8 \times 1^2)$   
 $d = 4.9\text{m}$

iv. They both hit the ground at the same time. Both experience the same constant acceleration due to gravity so they fall at the same rate.

d. i.  $d = \frac{v_i + v_f}{2}t$   
 $d = \frac{0.8 + 2.4}{2} \times 5.5$   
 $d = 8.8\text{m}$

ii.  $\sin(20) = \frac{y}{8.8}$   
 $y = 8.8\sin(20)$   
 $y = 3.01\text{m}$

iii.  $d = v_i t + \frac{1}{2}at^2$

Since the initial velocity is 0:

$$d = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2d}{a}}$$

$$t = \sqrt{\frac{2 \times 2.67}{9.8}}$$

$$t = 0.738\text{s}$$

iv.  $v_f = v_i + at$   
 $= 0 + 9.8 \times 0.738$   
 $= 7.23\text{ms}^{-1}$

v. After the chip hits the water it decelerates (accelerates upwards) due to the buoyancy force opposing gravity. Once the speed reaches zero, the chip continues to accelerate in the same direction until it reaches the surface, where the buoyancy force will reduce to equal the weight force (ignoring the slight bobbing the chip will do on the surface).

e. i.  $v_i = 50\text{km/h} \times \frac{1000}{60 \times 60} = 13.8\text{ms}^{-1}$   
 $v_i = 30\text{km/h} \times \frac{1000}{60 \times 60} = 8.3\text{ms}^{-1}$

ii.  $v_f = v_i + at$   
 $t = \frac{v_f - v_i}{a}$   
 $t = \frac{8.3 - 13.8}{-2.06}$   
 $t = 2.67\text{s}$

iii.  $d = \frac{v_i + v_f}{2}t$   
 $d = \frac{13.8 + 8.3}{2} \times 2.67$   
 $d = 29.50\text{m}$

The car will be 4.97m past the end of the roadworks when it reaches the speed limit.

$$iv. v_f^2 = v_i^2 + 2ad$$

$$v_f = \sqrt{v_i^2 + 2ad}$$

$$v_f = \sqrt{13.8^2 + 2 \times -2.06 \times 12.5}$$

$$v_f = 11.79 \text{ms}^{-1}$$

$$v_f = 11.79 \text{ms}^{-1} \frac{60 \times 60}{1000} = 42.43 \text{km/h}$$

The car is travelling  $42.43 - 30 = 12.43 \text{km/h}$  over the speed limit when it passes the camera.

$$f. i. v = 93 \times \frac{1000}{60 \times 60} = 25.83 \text{ms}^{-1}$$

$$ii. v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t}$$

$$a = \frac{25.83 - 0}{2.89}$$

$$a = 8.94 \text{ms}^{-2}$$

$$iii. v_f^2 = v_i^2 + 2ad$$

$$d = \frac{v_i^2 - v_f^2}{2a}$$

$$d = \frac{12^2 - 0}{2 \times 3.09}$$

$$d = 23.30 \text{m}$$

$$g. i. d = \frac{v_i + v_f}{2} t$$

$$d = \frac{0 + 61.7}{2} \times 30$$

$$d = 925.5 \text{m}$$

$$ii. d = v_i t + \frac{1}{2} at^2$$

$$d = (75 \times 20) + \left(\frac{1}{2} 1.5 \times 20^2\right)$$

$$d = 1800 \text{m}$$

$$\sin(15) = \frac{y}{1800}$$

$$y = 1800 \sin(15)$$

$$y = 465.87 \text{m}$$

$$h. i. v = 65 \times \frac{1000}{60 \times 60} = 18.06 \text{ms}^{-1}$$

$$ii. v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t}$$

$$a = \frac{0 - 18.06}{1.90}$$

$a = -9.51 \text{ms}^{-2}$ , the car accelerates in the direction opposite to its motion.

$$\begin{aligned}\text{iii. } d &= v_i t + \frac{1}{2} a t^2 \\ d &= (18.061 \times 90) + \left(\frac{1}{2} \times -9.51 \times 1.90^2\right) \\ d &= 17.15\text{m}\end{aligned}$$

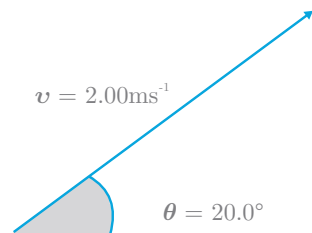
$$\text{Distance into intersection} = 17.15 - 16 = 1.15\text{m}$$

$$\begin{aligned}\text{iv. } d &= \frac{v_i + v_f}{2} t \\ v_i &= \frac{2d}{t} - v_f \\ v_i &= \frac{2 \times 15}{1.90} - 0 \\ v_i &= 15.78947368\text{ms}^{-1} \\ v_i &= 15.78947368 \times \frac{60 \times 60}{1000} = 56.84\text{km/h}\end{aligned}$$

## 7. Projectile Motion

### 1. Velocity Decomposition

a. i.



$$\sin(\theta) = \frac{O}{H} = \frac{v_y}{v}$$

$$v_y = \sin(\theta)v$$

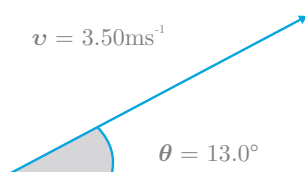
$$v_y = \sin(20) \times 2 = 0.684 \text{ ms}^{-1}$$

$$\cos(\theta) = \frac{A}{H} = \frac{v_x}{v}$$

$$v_x = \cos(\theta)v$$

$$v_x = \cos(20) \times 2 = 1.88 \text{ ms}^{-1}$$

ii.



$$\sin(\theta) = \frac{O}{H} = \frac{v_y}{v}$$

$$v_y = \sin(\theta)v$$

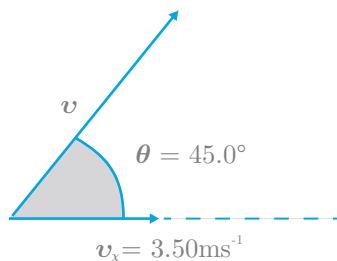
$$v_y = \sin(13) \times 3.5 = 0.787 \text{ ms}^{-1}$$

$$\cos(\theta) = \frac{A}{H} = \frac{v_x}{v}$$

$$v_x = \cos(\theta)v$$

$$v_x = \cos(13) \times 3.5 = 3.41 \text{ ms}^{-1}$$

b. i.



$$\tan(\theta) = \frac{O}{A} = \frac{v_y}{v_x}$$

$$v_y = \tan(\theta)v_x$$

$$v_y = \tan(45) \times 3.5 = 3.5 \text{ ms}^{-1}$$

$$v^2 = v_x^2 + v_y^2$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{3.5^2 + 3.5^2} = 4.95 \text{ ms}^{-1}$$

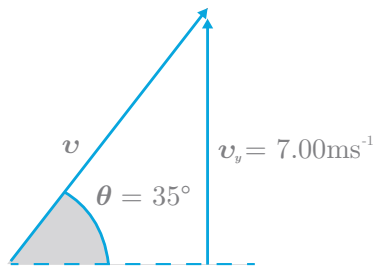
OR

$$\cos(\theta) = \frac{A}{H} = \frac{v_x}{v}$$

$$v = \frac{v_x}{\cos(\theta)}$$

$$v = \frac{3.5}{\cos(45)} = 4.95 \text{ ms}^{-1}$$

ii.



$$\tan(\theta) = \frac{O}{A} = \frac{v_y}{v_x}$$

$$v_x = \frac{v_y}{\tan(\theta)}$$

$$v_x = \frac{7}{\tan(35)} = 10.00 \text{ ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{10^2 + 7^2} = 12.21 \text{ ms}^{-1}$$

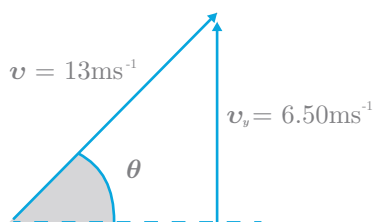
OR

$$\sin(\theta) = \frac{O}{H} = \frac{v_y}{v}$$

$$v = \frac{v_y}{\sin(\theta)}$$

$$v = \frac{7}{\sin(35)} = 12.20 \text{ ms}^{-1}$$

iii.



$$\sin(\theta) = \frac{O}{H} = \frac{v_y}{v}$$

$$\theta = \sin^{-1}\left(\frac{v_y}{v}\right)$$

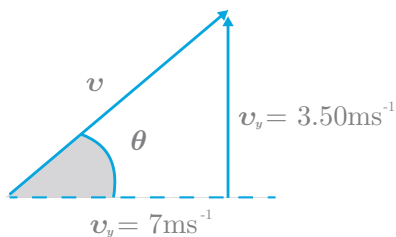
$$\theta = \sin^{-1}\left(\frac{6.5}{13}\right) = 30^\circ$$

$$v^2 = v_x^2 + v_y^2$$

$$v_x = \sqrt{v^2 - v_y^2}$$

$$v_x = \sqrt{13^2 - 6.5^2} = 11.26 \text{ ms}^{-1}$$

iv.



$$\tan(\theta) = \frac{O}{A} = \frac{v_y}{v_x}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

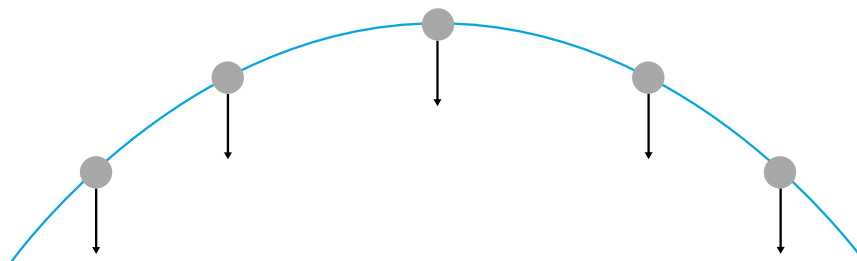
$$\theta = \tan^{-1}\left(\frac{3.5}{7}\right) = 26.57^\circ$$

$$v = \sqrt{v_x^2 + v_y^2}$$

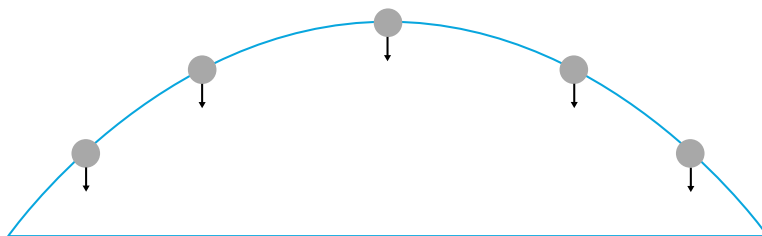
$$v = \sqrt{3.5^2 + 7^2} = 7.83 \text{ ms}^{-1}$$

2.

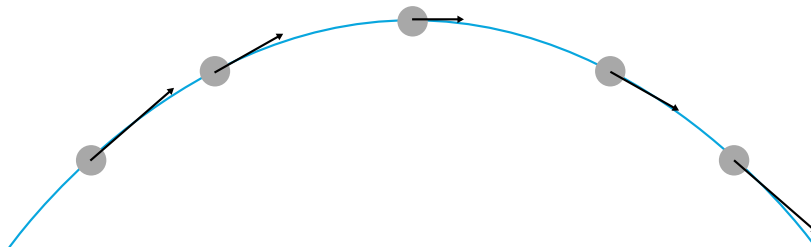
a.



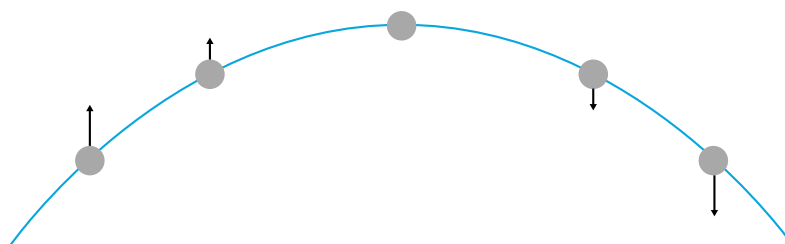
b.



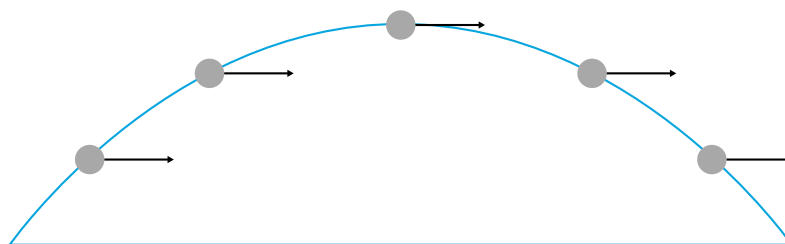
c. i.



ii.



iii.



- d. i. Nothing happens to the force and acceleration during the motion, since only the force of gravity is acting on the projectile (which always points downwards and does not change in size).
- ii. The vertical component of the velocity is initially large and points upwards, but decreases to zero then continues to decrease to become large and pointing downwards. The cause for this is the weight force since the weight force points downwards the whole time it generates an acceleration which points downwards the whole time. This causes the vertical component of the velocity to decrease throughout the motion.
- iii. Nothing happens to the horizontal velocity during the motion. This is because there are no forces pointing in the horizontal direction, only the weight force pointing straight downwards, which generates an acceleration which points straight downwards, which has no effect on the horizontal velocity. Since there is no acceleration in the horizontal direction, the horizontal velocity remains constant.



## 8. Circular Motion

- a. i. The car's speed is constant but its velocity isn't as its direction is changing. This means the car is experiencing acceleration towards the centre of the roundabout (at a right angle to the velocity of the car).

ii. The friction force acting on the car's wheels provides the centripetal force.

iii.  $v = 25 \times \frac{1000}{60 \times 60} = 6.94 \text{ms}^{-1}$

iv.  $F_c = \frac{mv^2}{r}$   
 $F_c = \frac{1260 \times 6.94^2}{0.5 \times 32.9}$   
 $F_c = 3689.13 \text{N}$

- v. As the car accelerates towards the centre of the roundabout, a passenger may feel like they are being pushed towards the outside of the corner. However, this is just their body wanting to continue on its current velocity and is not a real force. The force is instead the car keeping them going around the corner.

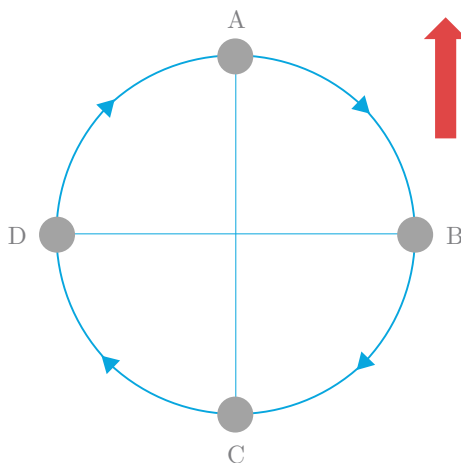
- b. i. The tension force in the wire.

ii.  $d = 2\pi r = 2\pi \times 1.7 = 10.68 \text{m}$

iii.  $v = \frac{d}{t}$   
 $v = \frac{10.68}{1.2}$   
 $v = 8.90 \text{ms}^{-1}$

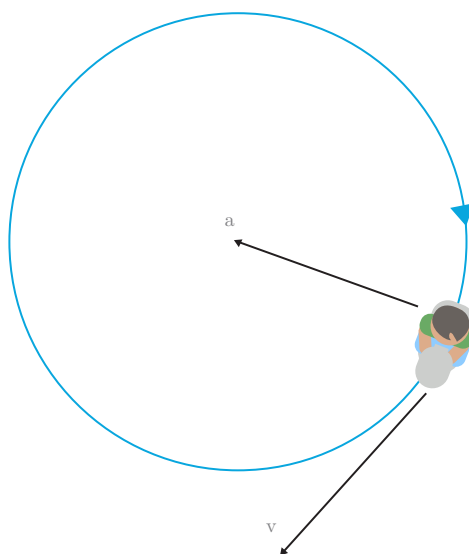
iv.  $F_c = \frac{mv^2}{r}$   
 $F_c = \frac{8 \times 8.90^2}{1.7}$   
 $F_c = 372.85 \text{N}$

v.



It will move in a straight line in the direction of its velocity at the time of release (tangential to the circle). He should release the weight at D because at this point the velocity is in the desired direction.

c. i., ii.



iii.  $d = 2\pi r = 2\pi \times 3.0 = 18.85\text{m}$

iv.  $v = \frac{\Delta d}{\Delta t}$   
 $v = \frac{18.85}{12.8}$   
 $v = 1.47\text{ms}^{-1}$

v.  $F_c = \frac{mv^2}{r}$   
 $F_c = \frac{26 \times 1.47^2}{3.0}$   
 $F_c = 18.73\text{N}$

d. i.  $C = 2\pi r$

$$r = \frac{C}{2\pi}$$

$$r = \frac{4 \times 13}{2\pi}$$

$$r = 8.28\text{m}$$

ii. Kerry's speed is constant but her velocity is changing because it is a vector quantity, meaning it has direction and magnitude. The magnitude of the vector stays the same, but the direction is changing due to a centripetal acceleration towards the centre of the circular path. The acceleration is provided by a net centripetal force which is provided by the friction force on the tires of the car.

$$iii. a_c = \frac{v^2}{r}$$

$$a_c = \frac{9.7^2}{8.28}$$

$$a_c = 11.36 \text{ms}^{-2}$$

$$iv. F_c = ma_c \text{ or } F_c = \frac{mv^2}{r}$$

$$m = \frac{F_c}{a_c} \text{ or } m = \frac{F_c r}{v^2}$$

$$m = \frac{15050}{11.36} \text{ or } m = \frac{15050 \times 8.28}{9.7^2}$$

$$m = 1324 \text{kg}$$

$$e. i. v = \frac{\Delta d}{\Delta t}$$

$$v = \frac{4 \times 2\pi r}{\Delta t}$$

$$v = \frac{8\pi \times 0.75}{3}$$

$$v = 6.28 \text{ms}^{-1}$$

$$ii. F_c = \frac{mv^2}{r}$$

$$F_c = \frac{0.060 \times 6.28^2}{0.75}$$

$$F_c = 3.16 \text{N}$$

iii. The centripetal force is provided by the tension force in the string. It causes an acceleration towards the centre of the circle which changes the direction but not the magnitude of the ball's velocity, meaning it travels in a circular path at a constant speed. The direction of the velocity is always tangential to the circle, so when Maia lets it go, it will continue in that direction.

iv.  $F_c = \frac{mv^2}{r}$  and  $v = \sqrt{\frac{rF_c}{m}}$ , so speed is proportional to the square root of the radius. That means if the radius decreases, so too does the speed.

$$f. i. F_g = mg$$

$$F_g = 500 \times 9.8$$

$$F_g = 4900 \text{N}$$

$$ii. F_c = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{F_c r}{m}}$$

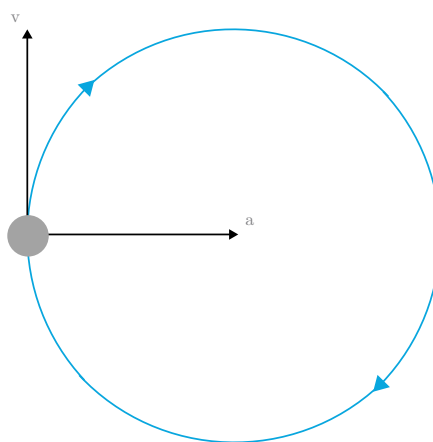
$$v = \sqrt{\frac{4900 \times (6.371 \times 10^6 + 2 \times 10^6)}{500}}$$

$$v = 9057.36 \text{ms}^{-1}$$

$$\begin{aligned}
 \text{iii. } v &= \frac{\Delta d}{\Delta t} \\
 t &= \frac{\Delta d}{v} \\
 t &= \frac{2\pi r}{v} \\
 t &= \frac{2\pi \times (6.371 \times 10^6 + 2 \times 10^6)}{9057.36} \\
 t &= 5807.05\text{s} \\
 t &= 1.61 \text{ hours}
 \end{aligned}$$

iv.  $F_c = \frac{mv^2}{r}$  and  $r = \frac{mv^2}{F_c}$ , so radius is proportional to the square of the speed. That means if the speed increases, the radius of the orbit must also increase.

g. i., ii.



$$\begin{aligned}
 \text{iii. } F_c &= \frac{mv^2}{r} \\
 F_c &= \frac{2.4 \times 10^2}{1.15} \\
 F_c &= 208.70\text{N}
 \end{aligned}$$

$$\begin{aligned}
 \text{iv. } d &= 2\pi r \\
 d &= 2\pi \times 1.15 \\
 d &= 7.23\text{m}
 \end{aligned}$$

v. Distance travelled in one rotation:

$$\begin{aligned}
 \Delta d &= v\Delta t \\
 \Delta d &= 10 \times 1.6 \\
 \Delta d &= 16\text{m}
 \end{aligned}$$

So in that time he completes  $\frac{16}{7.23} = 2.21$  rotations. So 2 full rotations in that time.

h. i.  $F_c = ma_c$

$$\begin{aligned}
 m &= \frac{F_c}{a_c} \\
 m &= \frac{228.4}{3.09} \\
 m &= 73.92\text{kg}
 \end{aligned}$$

$$ii. a_c = \frac{v^2}{r} \text{ or } F_c = \frac{mv^2}{r}$$

$$r = \frac{v^2}{a_c} \text{ or } r = \frac{mv^2}{F_c}$$

$$r = \frac{11^2}{30.9} \text{ or } r = \frac{73.92 \times 11^2}{228.4}$$

$$r = 39.16\text{m}$$

$$iii. d = 3 \times 2\pi r$$

$$d = 6\pi \times 39.16$$

$$d = 738.1217691\text{m}$$

$$v = \frac{\Delta d}{\Delta t}$$

$$t = \frac{d}{v}$$

$$t = \frac{738.1217691}{11}$$

$$t = 67.10\text{s}$$

- iv.  $F_c = \frac{mv^2}{r}$ , so force is inversely proportional to the radius. That means if the radius doubles, the force acting on the biker will halve.

## 9. Torque

a. i.  $\tau = Fd$   
 $\tau = 20 \times 3$   
 $\tau = 60\text{Nm}$ , anticlockwise

ii.  $\tau = Fd$   
 $\tau = 20 \times 6$   
 $\tau = 120\text{Nm}$ , clockwise

iii. Half the length of the handle is 15cm or 0.15m. So,  
 $\tau = Fd$   
 $\tau = 0.15 \times 30$   
 $\tau = 4.5\text{Nm}$ , clockwise

iv. Now that the wrench is turned with two hands we'll have to add up both the torques.  
 $\tau = Fd$   
 $\tau_1 = 0.15 \times 15, \quad \tau_2 = 0.3 \times 15$   
 $\tau = \tau_1 + \tau_2 = 2.25 + 4.5$   
 $\tau = 6.75\text{Nm}$ , clockwise

v. If we compare the torque exerted by the same force in questions iii) and iv) (both use a total of 30N), then holding the wrench further away from the turning point generates a greater torque, since when the wrench is held from the end and the middle (rather than just the middle), the torque increases. This is also illustrated in the formula  $\tau = Fd$ , increasing the distance  $d$  with the same force  $F$  increases the torque.

b. i.  $\tau = Fd$   
 $\tau_1 = 12 \times 3$   
 $\tau_1 = 36\text{Nm}$ , anticlockwise  
 $\tau_2 = 24 \times 6$   
 $\tau_2 = 144\text{Nm}$ , clockwise  
 $\tau_{\text{total}} = \tau_1 + \tau_2$   
 $\tau_{\text{total}} = 36 + 144 = 180\text{Nm}$ , anticlockwise

ii. First, convert to the standard units of metres. 60cm = 0.6m, 30cm = 0.3m.  
 $\tau = Fd$   
 $\tau_1 = 1.2 \times 0.3$   
 $\tau_1 = 0.36\text{Nm}$ , clockwise  
 $\tau_2 = 2.7 \times 0.6$   
 $\tau_2 = 1.62\text{Nm}$ , clockwise  
 $\tau_{\text{total}} = \tau_1 + \tau_2$   
 $\tau_{\text{total}} = 0.36 + 1.62 = 1.98$   
 $\tau_{\text{total}} = 1.98\text{Nm}$ , clockwise

iii. This is a bit of a weird one since the torques aren't all pointing in the same direction. That means that we're going to have to pick which ones are negative and which ones are positive, just like with forces that point in different directions. We're going to pick clockwise torques to be positive, but it doesn't matter which one you pick.

$$\begin{aligned}\tau &= Fd \\ \tau_1 &= 10 \times 3 \\ \tau_1 &= 30\text{Nm, anticlockwise} \\ \tau_2 &= 10 \times 4 \\ \tau_2 &= 40\text{Nm, clockwise} \\ \tau_{\text{total}} &= -\tau_1 + \tau_2 \\ \tau_{\text{total}} &= -30 + 40 = 10\text{Nm, clockwise}\end{aligned}$$

## 2.

a. i.  $\tau_{\text{clockwise}} = 10 \times 10$   
 $\tau_{\text{clockwise}} = 100\text{Nm}$   
 $\tau_{\text{anticlockwise}} = 2.5 \times F$

Since there is no net torque,

$$\begin{aligned}\tau_{\text{anticlockwise}} &= \tau_{\text{clockwise}} \\ 2.5F &= 100 \\ F &= 100/2.5 = 40\text{N, up}\end{aligned}$$

iii. This one is a little harder since there are more forces. As long as you make sure you know which ones are clockwise and anticlockwise, it's the same process!

$$\begin{aligned}\tau_{\text{clockwise}} &= 0.6 \times 5 + 0.3 \times F \\ \tau_{\text{clockwise}} &= 3 + 0.3F \\ \tau_{\text{anticlockwise}} &= 5 \times 0.3 \\ \tau_{\text{anticlockwise}} &= 1.5\text{Nm}\end{aligned}$$

Since there is no net torque,

$$\begin{aligned}\tau_{\text{clockwise}} &= \tau_{\text{anticlockwise}} \\ 3 + 0.3F &= 1.5 \\ 0.3F &= 1.5 - 3 = -1.5 \\ F &= -1.5/0.3 = -5\text{N, down}\end{aligned}$$

So, in this case, the force is negative. Therefore, it should actually be pointing upwards! Don't worry, we can just make it positive and pointing in the opposite direction, which gives us:

$$F = 5\text{N, up}$$

iv. For this one, we're going to pick clockwise torques as positive again.

$$\begin{aligned}\tau &= Fd \\ \tau_1 &= 0.75 \times 20 \\ \tau_1 &= 15\text{Nm, anticlockwise} \\ \tau_2 &= 10 \times 1 \\ \tau_2 &= 10\text{Nm, clockwise}\end{aligned}$$

$$\begin{aligned}\tau_{\text{total}} &= -15 + 10 \\ \tau_{\text{total}} &= -5\text{Nm, clockwise}\end{aligned}$$

That's a bit of a weird one since the torque came out negative, but just like with forces, a torque which is negative in one direction is positive in the opposite direction, so we could say:

$$\tau_{\text{total}} = 5\text{Nm, anticlockwise}$$

ii.  $\tau_{\text{anticlockwise}} = 10 \times 0.3$   
 $\tau_{\text{anticlockwise}} = 3\text{Nm}$   
 $\tau_{\text{clockwise}} = 0.6 \times F$

Since there is no net torque,

$$\begin{aligned}\tau_{\text{anticlockwise}} &= \tau_{\text{clockwise}} \\ 0.6F &= 3 \\ F &= 3/0.6 = 5\text{N, up}\end{aligned}$$

iv.  $\tau_{\text{clockwise}} = 7.5 \times 7 + 30 \times 3.5$   
 $\tau_{\text{clockwise}} = 157.5\text{Nm}$   
 $\tau_{\text{anticlockwise}} = F \times 7 + 15 \times 3.5$   
 $\tau_{\text{anticlockwise}} = 7F + 52.5$

Since there is no net torque,

$$\begin{aligned}\tau_{\text{clockwise}} &= \tau_{\text{anticlockwise}} \\ 157.5 &= 7F + 52.5 \\ 157.5 - 52.5 &= 7F \\ F &= 105/7 \\ F &= 15\text{N, up}\end{aligned}$$

- b. i. We're going to choose A as the support because, if you choose B, you remove the force that we're trying to find from the equation. Notice how we don't count the force from point A - that's because it's 0m from the pivot, so it doesn't produce a torque.

$$\tau_{\text{clockwise}} = 3 \times 3 + F \times 6$$

$$\tau_{\text{clockwise}} = 9 + 6F$$

$$\tau_{\text{anticlockwise}} = 6 \times (3+6)$$

$$\tau_{\text{anticlockwise}} = 54\text{Nm}$$

Since there are two supports involved and supports don't move, the system must be in rotational equilibrium, so:

$$\tau_{\text{clockwise}} = \tau_{\text{anticlockwise}}$$

$$9 + 6F = 54$$

$$6F = 45$$

$$F = 45/6$$

$$F = 7.5\text{N, down}$$

- ii. We're going to choose B to be the pivot for this one, but you could also choose A and get the same result.

$$\tau_{\text{clockwise}} = 3 \times 8 + 4 \times F$$

$$\tau_{\text{clockwise}} = 24 + 4F$$

$$\tau_{\text{anticlockwise}} = 10.625 \times (4 + 4)$$

$$\tau_{\text{anticlockwise}} = 85\text{Nm}$$

$$\tau_{\text{clockwise}} = \tau_{\text{anticlockwise}}$$

$$24 + 4F = 85$$

$$4F = 61$$

$$F = 15.25\text{N, down}$$



## 10. Momentum

- a. i. The total momentum of the system is conserved.

$$\begin{aligned} \text{ii. } p_A &= m_A v_A \\ p_A &= 55 \times 5.0 \\ p_A &= 275 \text{ kgms}^{-1} \end{aligned}$$

$$\begin{aligned} \text{iii. } p_B &= m_B v_B \\ p_B &= 40 \times 3.5 \\ p_B &= 140 \text{ kgms}^{-1} \end{aligned}$$

$$\begin{aligned} \text{iv. } p_i &= p_A + p_B \\ p_i &= 275 + 140 \\ p_i &= 415 \text{ kgms}^{-1} \end{aligned}$$

$$\text{v. } p_f = p_i = 415 \text{ kgms}^{-1}$$

$$\text{vi. } (m_A + m_B)v_f = p_f$$

$$v_f = \frac{p_f}{m_A + m_B}$$

$$v_f = \frac{415}{55 + 40}$$

$$v_f = 4.37 \text{ ms}^{-1}$$

$$\begin{aligned} \text{vii. } \Delta p &= p_{Bf} - p_{Bi} \\ \Delta p &= (40 \times 4.37) - 140 \\ \Delta p &= 34.8 \text{ ms}^{-1} \end{aligned}$$

$$\Delta p = F \Delta t$$

$$F = \frac{\Delta p}{\Delta t}$$

$$F = \frac{34.8}{2.0}$$

$$F = 17.4 \text{ N}$$

- b. i. During the collision, both cars experience a force exerted by the other car. The stationary car moves forward while the other car is slowed down by the force acting against its motion.

$$\text{ii. } v = 10 \times \frac{1000}{60 \times 60} = 2.7 \text{ ms}^{-1}$$

$$\begin{aligned} \text{iii. } p &= mv \\ p &= 1200 \times 2.7 \\ p &= 3333.3 \text{ kgms}^{-1} \end{aligned}$$

$$\text{iv. } p_f = 1200 \times 0.88$$

$$p_f = 1056 \text{ kgms}^{-1}$$

$$\Delta p = p_f - p_i$$

$$\Delta p = 1056 - 3333.3$$

$$\Delta p = -2277.3 \text{ kgms}^{-1}$$

$$\text{v. } \Delta p = F \Delta t$$

$$F = \frac{\Delta p}{\Delta t}$$

$$F = - \frac{-2277.3}{0.7}$$

$$F = -3253.3 \text{ N}$$

The force is negative because it's acting in the opposite direction to the motion of the car.

vi. The total momentum of the system is conserved, assuming no external forces act on the system, so:

$$p_{\text{parked car}} = p_{\text{moving car before}} - p_{\text{moving car after}}$$

$$p_{\text{parked car}} = 3333.3 - 1056$$

$$p_{\text{parked car}} = 2277.3 \text{ kgms}^{-1}$$

$$p = mv$$

$$v = \frac{p}{m}$$

$$v = \frac{2277.3}{31200}$$

$$v = 1.90 \text{ ms}^{-1}$$

$$\text{c. i. } \Delta p = F \Delta t$$

$$\Delta t = \frac{\Delta p}{F}$$

$$\Delta t = \frac{11.62}{166}$$

$$\Delta t = 0.07 \text{ s}$$

$$\text{ii. } p = mv$$

$$v = \frac{p}{m}$$

$$v = \frac{11.62}{0.415}$$

$$v = 28 \text{ ms}^{-1}$$

iii. A more deflated ball is softer and so the collision time for the kick is increased. Since  $F = \frac{\Delta p}{\Delta t}$  the force on the ball is decreased.

iv.  $p = mv$

$$p = (2 \times 0.415) \times 28$$

$$p = 23.24 \text{ kgms}^{-1}$$

$$\Delta p = F \Delta t$$

$$F = \frac{\Delta p}{\Delta t}$$

$$F = \frac{23.24}{0.07}$$

$$F = 33.2 \text{ kgms}^{-1}$$

d. i.  $p_{Ai} = m_A v_{Ai}$

$$p_{Ai} = 3.20 \times 4.0$$

$$p_{Ai} = 12.8 \text{ kgms}^{-1}$$

$$p_{Bi} = m_B v_{Bi}$$

$$p_{Bi} = 5.0 \times 5.4$$

$$p_{Bi} = 27 \text{ kgms}^{-1}$$

ii. Assuming no external forces act on the system, total momentum of the system is conserved so:

$$p_f = p_{Ai} + p_{Bi}$$

$$p_f = 12.8 + 27 = 39.8 \text{ kgms}^{-1}$$

$$(m_A + m_B)v_f = p_f$$

$$v_f = \frac{p_f}{m_A + m_B}$$

$$v_f = \frac{39.8}{3.20 + 5.0}$$

$$v_f = 4.85 \text{ ms}^{-1}$$

iii.  $\Delta p = p_{Af} - p_{Ai}$

$$\Delta p = (3.2 \times 4.85) - 12.8$$

$$\Delta p = 2.72 \text{ kgms}^{-1}$$

$$\Delta p = F \Delta t$$

$$\Delta t = \frac{\Delta p}{F}$$

$$\Delta t = \frac{2.72}{2.44}$$

$$\Delta t = 1.11 \text{ s}$$

iv. During the collision, the larger weight (B) hits the lighter, slower one (A), which causes A's speed to increase and B's to decrease. Because B has a larger mass, the final velocity of the two weights is closer to B's speed than A's. If the weights were reversed, the final velocity would have been closer to A's, so the overall motion would be slower.

e. i.  $p_{ii} = m_i v_{ii}$

$$p_{ii} = 2000 \times 10$$

$$p_{ii} = 20000 \text{ kgms}^{-1} \text{ Eastwards}$$

$$\begin{aligned}
 \text{ii. } p_{2i} &= m_2 v_{2i} \\
 p_{2i} &= 1600 \times 8 \\
 p_{2i} &= 12800 \text{ kgms}^{-1} \text{ Westwards}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii. } p_i &= 20000 - 12800 \\
 p_i &= 7200 \text{ kgms}^{-1} \text{ Eastwards}
 \end{aligned}$$

$$\text{iv. } p_f = 7200 \text{ kgms}^{-1} \text{ Eastwards}$$

$$\text{v. } (m_1 + m_2)v_f = p_f$$

$$v_f = \frac{p_f}{m_1 + m_2}$$

$$v_f = \frac{7200}{2000 + 1600}$$

$$v_f = 2 \text{ ms}^{-1}$$

vi. The airbags and bonnet crumpling increases the collision time. Since the same momentum is transferred, a longer collision time decreases the force of the collision, as  $F = \frac{\Delta p}{\Delta t}$ . This in turn reduces the damage to the car and the people inside.

$$\begin{aligned}
 \text{f. i. } p_{Di} &= m_D v_{Di} \\
 p_{Di} &= 72 \times 1.3 \\
 p_{Di} &= 93.6 \text{ kgms}^{-1} \text{ Rightwards}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii. } p_{Df} &= m_D v_{Df} \\
 p_{Df} &= 72 \times 2 \\
 p_{Df} &= 144 \text{ kgms}^{-1} \text{ Leftwards}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii. } p_i &= p_{Di} + p_{Ci} \\
 p_i &= 93.6 - 2.4m_C
 \end{aligned}$$

$$\begin{aligned}
 \text{iv. } p_f &= p_{Df} + p_{Cf} \\
 p_f &= -144 + 0m_C \\
 p_f &= -144 \text{ kgms}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{v. } p_i &= p_f \\
 93.6 - 2.4m_C &= -144 \\
 m_C &= \frac{-144 - 93.6}{-2.4} \\
 m_C &= 99 \text{ kg}
 \end{aligned}$$

vi. We will continue to treat rightwards momentum as positive.

$$\begin{aligned}
 \Delta p &= p_{Di} - p_{Df} \\
 \Delta p &= 93.6 - (-144) \\
 \Delta p &= 237.6 \text{ kgms}^{-1}
 \end{aligned}$$

$$\text{vii. } \Delta p = F \Delta t$$

$$\Delta t = \frac{\Delta p}{F}$$

$$\Delta t = \frac{237.6}{1248}$$

$$\Delta t = 0.190 \text{ s}$$

viii. If Cara were travelling slower, the momentum transferred to Dan would have decreased. Since the collision time is assumed to be the same, and  $F = \frac{\Delta p}{\Delta t}$ , the force experienced by Dan would have also decreased.

g. i.  $p = mv$

$$v = \frac{p}{m}$$

$$v = \frac{248}{56}$$

$$v = 4.43\text{ms}^{-1}$$

ii. If she bends her knees as she lands, Jane's stopping time will be increased which reduces the force, since the change in momentum is the same ( $\Delta p = F\Delta t$ ).

iii. Momentum is not conserved. Gravity acts as an external force on Jane as she falls, which increases her momentum.

h. i.  $p = mv$

$$p = 500 \times 83$$

$$p = 41.5\text{kgms}^{-1}$$

ii.  $\Delta p = F\Delta t$

$$F = \frac{\Delta p}{\Delta t}$$

$$F = \frac{41.5}{0.11}$$

$$F = 377.27\text{N}$$

iii. Removing the padding would decrease the stopping time of the fist, and since  $F = \frac{\Delta p}{\Delta t}$ , and the momentum transferred remains the same, this increases the force.

iv. Momentum is conserved. There are no external forces acting on the system.

## 11. Springs

a. i.  $F = mg$

$$F = 0.5 \times 9.8$$

$$F = 4.9\text{N}$$

ii.  $F = kx$

$$k = \frac{F}{x}$$

$$k = \frac{4.9}{0.1}$$

$$k = 49\text{Nm}^{-1}$$

iii. This would double the force acting on the spring. Since  $x = \frac{F}{k}$ , and  $k$  is constant, a doubled force would mean the extension of the spring would also double.

iv.  $E_p = \frac{1}{2}kx^2$

$$E_p = \frac{1}{2} \times 49 \times 10^2$$

$$E_p = 2450\text{J}$$

v.  $E_p = \frac{1}{2}kx^2$

$$E_p = \frac{1}{2} \times 49 \times 20^2$$

$$E_p = 9800\text{J}$$

4× as much energy is stored in the spring with two weights.

b. i.  $F = kx$

$$mg = kx$$

$$m = \frac{kx}{g}$$

$$m = \frac{18.7 \times 0.22}{9.8}$$

$$m = 0.420\text{kg}$$

ii.  $E_p = \frac{1}{2}kx^2$

$$E_p = \frac{1}{2} \times 18.7 \times 0.22^2$$

$$E_p = 0.453\text{J}$$

iii. If the energy were doubled, the cord would extend by:

$$E_p = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{2E_p}{k}}$$

$$x = \sqrt{\frac{2 \times (2 \times 0.453)}{18.7}}$$

$$x = 0.3113\text{m}$$

$$F = kx$$

$$F = 18.7 \times 0.3113$$

$$F = 5.82\text{N}$$

c. i.  $F = kx$

$$x = \frac{F}{k}$$

$$x = \frac{ma}{k}$$

$$x = \frac{30 \times 9.8}{5500}$$

$$x = 0.0535$$

ii.  $E_p = \frac{1}{2}kx^2$

$$E_p = \frac{1}{2} \times 5500 \times 0.0535^2$$

$$E_p = 7.87\text{J}$$

iii. The stiffness of the spring is due to the spring constant – the higher the spring constant, the stiffer the spring. A higher spring constant reduces the compression from the same force and reduces the energy stored in the spring.

d. i.  $F = kx$

$$k = \frac{F}{x}$$

$$x = \frac{ma}{k}$$

$$k = \frac{45 \times 9.8}{0.06}$$

$$k = 7350\text{Nm}^{-1}$$

ii.  $E_p = \frac{1}{2}kx^2$

$$E_p = \frac{1}{2} \times 7350 \times 0.06^2$$

$$E_p = 13.23\text{J}$$

iii.  $E_p = \frac{1}{2}kx^2$

$$x = \sqrt{\frac{2E_p}{k}}$$

$$x = \sqrt{\frac{2 \times 210}{7350}}$$

$$x = 0.239\text{m}$$

iv. Conservation of energy can be used. The elastic potential energy stored in the board must have come from the kinetic energy of the falling diver. So, we know the kinetic energy just before they land, because it's the same as the potential energy from before, and therefore can find their speed.

v.  $E_k = E_p = 210\text{J}$

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2 \times 210}{45}}$$

$$v = 3.06\text{ms}^{-1}$$

e. i.  $E_p = \frac{1}{2} kx^2$

$$E_p = \frac{1}{2} \times 145 \times 0.08^2$$

$$E_p = 0.464\text{J}$$

ii. Conservation of energy can be used. The elastic potential energy will be converted to kinetic energy, which can be used to calculate the band's speed.

iii.  $E_k = E_p = 0.464\text{J}$

$$E_k = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2 \times 0.464}{0.003}}$$

$$v = 17.59\text{ms}^{-1}$$

iv. We assumed that all of the elastic potential energy is converted to kinetic energy, and none is lost to heat, friction, etc.

f. i.  $E_p = \frac{1}{2} kx^2$

$$k = \frac{2E_p}{x^2}$$

$$k = \frac{2 \times 80}{0.15^2}$$

$$k = 7111.1\text{Nm}^{-1}$$

ii.  $F = kx$

$$F = 7111.1 \times 0.15$$

$$F = 1066.6\text{N}$$

iii. The elastic potential energy in the spring is converted into kinetic energy as the seat springs back.

iv. A stiffer spring has a higher spring constant which means it takes more force to achieve the same compression. John sitting on it would compress the spring, but not as much as the spring from the previous example.

g. i.  $F_g = \frac{1}{12} mg$

$$F_g = \frac{1}{12} \times 63 \times 9.8$$

$$F_g = 51.45\text{N}$$

ii.  $F = kx$

$$k = \frac{F}{x}$$

$$k = \frac{51.45}{0.053}$$

$$k = 970.75\text{Nm}^{-1}$$



$$\text{iii. } E_p = \frac{1}{2} kx^2$$

$$E_p = \frac{1}{2} \times 970.75 \times 0.053^2$$

$$E_p = 1.36\text{J}$$

- iv. Kate's mass would be spread across more springs so the force on each one would be lowered. This means each spring compresses less ( $F = kx$ ,  $k$  is constant), which decreases the amount of energy stored in them ( $E_p = \frac{1}{2} kx^2$ ,  $k$  is constant).

- h. The spring constant can be calculated from the gradient, since  $F = kx$  is in the form  $y = mx + c$ .

$$k = \frac{\text{rise}}{\text{run}} = \frac{5}{0.01} = 500\text{Nm}^{-1}$$

## 12. Work and Energy

- a. i. Energy cannot be created or destroyed. It can only be transferred or transformed.

ii.  $E_k = \frac{1}{2}mv^2$ , m stands for mass and v stands for speed.

iii.  $\Delta E_p = mg\Delta h$ , m stands for mass, g stands for the gravitational acceleration, and  $\Delta h$  stands for the change in height between two points.

iv.  $E_p = \frac{1}{2}kx^2$ , k stands for the spring constant and x stands for the displacement from equilibrium.

b.  $E_p = mgh = 12 \times 9.8 \times 10 = 1176\text{J}$

c. i.  $E_p = mgh = 12 \times 9.8 \times 5 = 588\text{J}$

- ii. Since energy is conserved, the total energy at this height must be the total energy the object started off with, so  $E_{\text{total}} = 1176$  – keep it unrounded until you give a final answer!

$$E_{\text{total}} = E_p + E_k$$

$$1176 = 588 + E_k$$

$$E_k = 588\text{J}$$

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2 \times 588}{12}}$$

$$v = 9.90\text{ms}^{-1}$$

d. i.  $E_p = mgh$

$$h = \frac{E_p}{mg}$$

$$h = \frac{100}{12 \times 9.8}$$

$$h = 0.850\text{m}$$

ii.  $E_p = 100\text{J}$

$$E_T = E_p + E_K = 1176$$

$$E_K + 100 = 1176$$

$$E_K = 1076\text{J}$$

$$\begin{aligned} \text{iii. } E_k &= \frac{1}{2} mv^2 \\ v &= \sqrt{\frac{2E_k}{m}} \\ v &= \sqrt{\frac{2 \times 1076}{12}} \\ v &= 13.39 \text{ms}^{-1} \end{aligned}$$

- e. Just before the object hits the ground, all its kinetic energy has been converted into potential energy. This means that  $E_k = 1176$ , so

$$\begin{aligned} E_k &= \frac{1}{2} mv^2 \\ v &= \sqrt{\frac{2E_k}{m}} \\ v &= \sqrt{\frac{2 \times 1176}{12}} \\ v &= 14 \text{ms}^{-1} \end{aligned}$$

- f. The work done on the object is really just the amount of energy that is converted. So, since all the 1176J of the object initially had as potential energy is converted to kinetic energy, the amount of energy that is converted is 1176J. This means the work done is  $W = 1176\text{J}$ . The weight force does this work.

$$\begin{aligned} \text{g. } P &= \frac{W}{t} \\ P &= \frac{1176}{1.43} \\ P &= 822.38 \text{W} \end{aligned}$$

## Section Two

# Exam Skills & Mixed Practice

### Linear Motion

1.

a. i.  $22\text{km/h} = 22 \times \frac{1000}{60^2} = 6.1\text{ms}^{-1}$

$$100\text{km/h} = 100 \times \frac{1000}{60^2} = 27.7\text{ms}^{-1}$$

$$68\text{km/h} = 68 \times \frac{1000}{60^2} = 18.8\text{ms}^{-1}$$

$$36\text{km/h} = 36 \times \frac{1000}{60^2} = 10\text{ms}^{-1}$$

ii. 1.  $v_i = 0\text{ms}^{-1}$ ,  $v_f = 6.1\text{ms}^{-1}$ ,  $t = 14\text{s}$

2.  $v_f = v_i + at$

3.  $v_f = v_i + at$

$$a = \frac{(v_f - v_i)}{t}$$

$$a = \frac{(6.1 - 0)}{14}$$

$$a = 0.436\text{ms}^{-2}$$

iii. 1.  $v_i = 6.1\text{ms}^{-1}$ ,  $v_f = 27.7\text{ms}^{-1}$ ,  $a = 0.437\text{ms}^{-2}$

2.  $v_f^2 = v_i^2 + 2ad$

3.  $v_f^2 = v_i^2 + 2ad$

$$d = \frac{v_f^2 - v_i^2}{2a}$$

$$d = \frac{27.7^2 - 6.1^2}{2 \times 0.436}$$

$$d = 837.25\text{m}$$

b. i. We will use conservation of momentum. To use this, we assume there are no external forces acting on the system.

ii.  $p_i = mv_i$

$$p_i = 1080 \times 27.7$$

$$p_i = 30,000\text{kgms}^{-1}$$

iii.  $p_f = mv_f$

$$p_f = 1080 \times 10$$

$$p_f = 10,800\text{kgms}^{-1}$$

iv.  $\Delta p = p_f - p_i$   
 $\Delta p = 10800 - 30000 = -19,200 \text{ kgms}^{-1}$   
 $\Delta p = 19,200 \text{ kgms}^{-1}$  in the opposite direction to the car's motion.

v.  $\Delta p = F \Delta t$

$$F = \frac{\Delta p}{\Delta t}$$

$$F = \frac{19200}{1.2}$$

$$F = 16,000 \text{ N}$$

c. i. It increases the time of the collision.

ii.  $F = \frac{\Delta p}{\Delta t}$ . The same change in momentum occurring over a longer time results in a lower force, which means less damage to the rest of the car and the people inside.

2.

a. i. The skydiver experiences a net force downwards causing them to accelerate as the force of gravity outweighs the opposing force from air resistance. As they approach terminal velocity, the opposing force increases.

ii. The skydiver experiences no net force as the gravity and air resistance forces are balanced, meaning that they don't accelerate and fall at a constant speed.

iii. When the skydiver opens their parachute, the air resistance increases. Because of this, there is a net force upwards, causing them to decelerate quickly.

b. i.  $a = \frac{\Delta v}{\Delta t}$

$$\Delta v = a \Delta t$$

$$\Delta v = 9.8 \times 12 = 117.6 \text{ ms}^{-1}$$

Since they started from rest, their final speed would be  $117.6 \text{ ms}^{-1}$ .

ii.  $d = v_i t + \frac{1}{2} a t^2$

iii.  $d = v_i t + \frac{1}{2} a t^2$

$$d = 117.6 \times 3.5 + \frac{1}{2} (-31.3) \times 3.5^2$$

$$d = 219.89 \text{ m}$$

c. i.  $v_f = v_i + at$

$$v_f = 117.6 + (-31.3) \times 3.5$$

$$v_f = 8.05 \text{ ms}^{-1}$$

ii.  $E_{ki} = \frac{1}{2} m v^2$

$$E_{ki} = \frac{1}{2} \times 55 \times 117.6^2$$

$$E_{ki} = 380,318.4 \text{ J}$$

iii.  $E_{\text{kf}} = \frac{1}{2} mv^2$

$$E_{\text{kf}} = \frac{1}{2} \times 55 \times 8.05^2$$

$$E_{\text{kf}} = 1,782.07\text{J}$$

iv.  $\Delta E_{\text{k}} = E_{\text{kf}} - E_{\text{ki}}$

$$\Delta E_{\text{k}} = 1,782.07 - 380,318.4$$

$$\Delta E_{\text{k}} = -378,536.33\text{J}$$

The skydiver loses 378,536.33J of energy.

## Circular Motion

1.

- a. i. Velocity is a vector quantity, which means that it has a size as well as a direction. Meanwhile, speed is a scalar quantity, so only has a size. Acceleration is a change in velocity, not speed.

ii The acceleration is a change in the direction of the object's velocity, rather than the size. Since speed is a scalar, this means the speed is unchanged.

- b. i. Speed of red car:

$$v_r = \frac{d}{t}$$

$$v_r = \frac{2\pi r}{T}$$

$$v_r = \frac{2\pi \times 13.5}{6.0} = 14.14\text{ms}^{-1}$$

Speed of blue car:

$$v_b = \frac{d}{t}$$

$$v_b = \frac{2\pi r}{T}$$

$$v_b = \frac{2\pi \times 11.0}{6.0} = 11.52\text{ms}^{-1}$$

- ii. Centripetal force on red car:

$$F_{cr} = \frac{mv^2}{r}$$

$$F_{cr} = \frac{1520 \times 14.14^2}{13.5}$$

$$F_{cr} = 22,511.70\text{N}$$

Centripetal Force on blue car:

$$F_{cb} = \frac{mv^2}{r}$$

$$F_{cb} = \frac{2180 \times 11.52^2}{11.0}$$

$$F_{cb} = 26,300.8\text{N}$$

- iii. Red car revolutions:

$$F_{cr} - F_{cb} = 22511.7 - 26300.8$$

$$= -3789.1$$

So the force experienced by the red car is 3789.1N less than the blue car.

- c. i. For both cars:

$$v = \frac{\Delta d}{\Delta t}$$

$$\Delta d = v\Delta t$$

$$\Delta d = 7 \times 60$$

$$\Delta d = 420\text{m}$$

ii. Red car:

$$d_r = 2\pi r$$

$$d_r = 2\pi \times 13.5 = 84.82\text{m}$$

Blue car:

$$d_b = 2\pi r$$

$$d_b = 2\pi \times 11.0 = 69.12\text{m}$$

iii. Red car revolutions:

$$\text{rev}_r = \frac{420}{84.82}$$

$$\text{rev}_r = 4.95$$

Blue car revolutions:

$$\text{rev}_b = \frac{420}{69.12}$$

$$\text{rev}_b = 6.08$$

iv. Difference =  $6.08 - 4.95 = 1.13$

The blue car completes 11 revolutions more than the red car.

2.

a. The velocity of the weight is constant in magnitude but its direction is changing. It's always tangential to the circular path, so in order to throw the weight forwards, Jared needs to release it at the point where its velocity is directed forward. So he should release it at position A.

b. i.  $C = 2\pi r$

$$C = 2\pi \times 2.0 = 12.57\text{m}$$

ii.  $T = \frac{\text{time}}{\text{revolutions}}$

$$T = \frac{2}{3}$$

$$T = 0.6 \text{ seconds per revolution}$$

iii.  $v = \frac{d}{t}$

$$v = \frac{12.57}{0.6}$$

$$v = 20.95\text{ms}^{-1}$$

iv.  $F_c = \frac{mv^2}{r}$

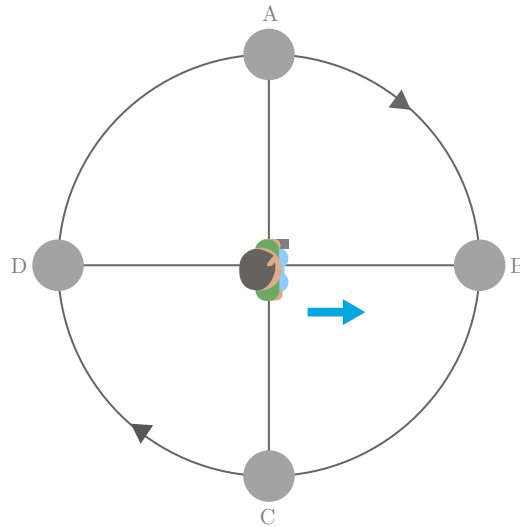
$$m = \frac{F_c r}{v^2}$$

$$m = \frac{1420 \times 2.0}{20.95^2}$$

$$m = 6.47\text{kg}$$



c. i.



ii. Velocity is a vector quantity, which means that it has a size as well as a direction. Meanwhile speed is a scalar quantity, so only has a size. Acceleration is a change in velocity, not speed.

iii. Net force.

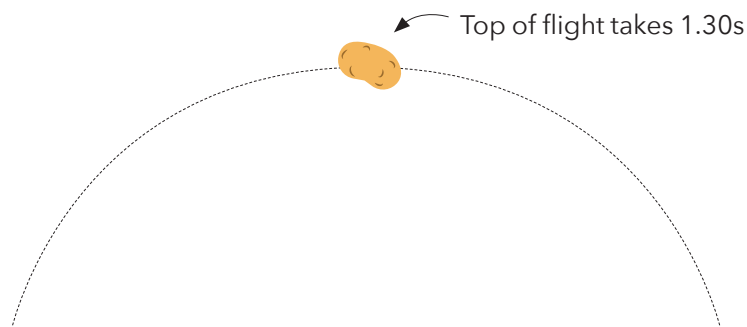
iv. The centripetal force provides an acceleration towards the centre of the circle. Acceleration is a change in velocity, not speed, and the acceleration changes the direction of the weight's velocity, but not the magnitude. Since speed is a scalar, it therefore remains constant.

v. The tension force in the steel wire.

## Projectile Motion

1.

- a. i. The highest point is in the middle of the flight. Because it takes 2.60 seconds to reach the ground, it will take 1.30s to reach the top of its flight, also called the apex.



- ii. The only force that will be acting on the potato is gravity, which always acts downwards. This means that the acceleration of the potato is  $9.8\text{ms}^{-2}$  downwards.
- iii. The vertical component at the highest point in the flight is going to be  $0\text{ms}^{-1}$  because the potato was moving upwards and is now about to move downwards.
- iv. The question has told us time (to zero velocity), the final velocity (at the top of the flight) and the acceleration on the potato:

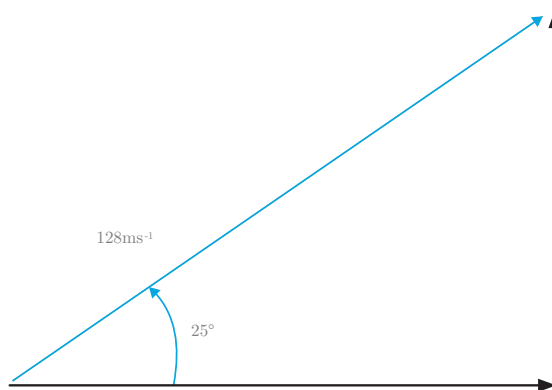
$$t = 1.3\text{s}, v_f = 0, v_i = ?, a = 9.8\text{ms}^{-2}, \theta = 25^\circ$$

The equation to use is  $v_f = v_i + at$  because it has three of the given quantities and one unknown.

$$\begin{aligned} \text{v. } v_i &= v_f - at \\ v_i &= 0 - (-9.8)(1.30) \\ v_i &= 12.74\text{ms}^{-1} \end{aligned}$$

$$\begin{aligned} \text{vi. } \sin(25^\circ) &= \frac{O}{H} \\ v_i &= \frac{v_i(\text{vertical})}{\sin(25^\circ)} \\ v_i &= \frac{12.74}{\sin(25^\circ)} \\ v_i &= 30.15\text{ms}^{-1} \end{aligned}$$

b. i.



Drawing the horizontal and vertical vectors of the given velocity will help us figure out which trig formula we need to use.

The initial vertical velocity is

$$v_{i(\text{vert})} = 128\sin(25^\circ) = 54.10\text{ms}^{-1}$$

The initial horizontal velocity is

$$v_{i(\text{horz})} = 128\cos(25^\circ) = 116.01\text{ms}^{-1}$$

We will need to use the horizontal velocity to calculate horizontal distance, but we also need the vertical velocity to find the time taken to fly (which is the same regardless of what direction we are thinking about).

- ii. We need to use vertical velocity to figure out the time it takes to travel. For this we first work out how long it takes to reach the top of its flight (when vertical velocity is 0). The information we have is:

$$v_i = 54.10\text{ms}^{-1}, v_f = 0\text{ms}^{-1}, a = -9.8\text{ms}^{-2}, t = ?$$

So we use the equation  $v_f = v_i + at$  and rearrange for time:

$$0 = 54.10 - 9.8t$$

$$t = \frac{-54.10}{-9.8} \quad 54.10 - 9.8 = 5.52\text{s}$$

Which is the time taken to reach halfway. So the full time is 11.04s.

- iii. So we use the horizontal velocity  $116.007\text{ms}^{-1}$  and the time it travels at this velocity to calculate the distance.

$$v = \frac{d}{t}$$

$$d = vt$$

$$d = 116.01 \times 11.04 = 1280.75\text{m}$$

c. i.  $v = \frac{d}{t}$

$$d = vt = 27.3 \times 2.60 = 71.0\text{m}$$

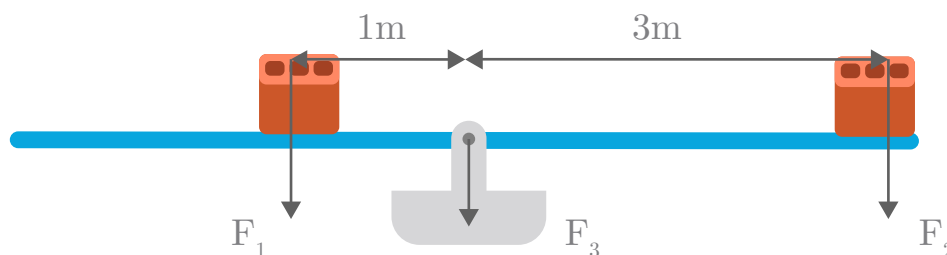
- ii. The second potato, fired by acetylene, travelled  $1280.75 - 71 = 1209.75\text{m}$  more than the spud fired using butane!

- d.
  - i. Both potatoes only experience the force of gravity acting on them during the flight, which causes the downwards acceleration that they both experience. Both potatoes have no horizontal force acting on them throughout the flight, so the horizontal velocity remains unchanged.
  - ii. Both potatoes exit at the same angle, but the one launched with acetylene has a much higher exit velocity. The horizontal velocities of both remain constant, while both vertical velocities decrease until they reach 0 at the peak of each of their arcs before increasing in the opposite direction. However, the acetylene potato reaches its peak much later, meaning that it reaches the ground much later and lands much further away.
- e.
  - i. In the equation  $v_f = v_i + at$ , which is used to calculate the time taken for the vertical velocity to reach  $0\text{ms}^{-1}$ ,  $-g$  is used as the acceleration.
  - ii. The time taken to reach the top of the arc will increase.
  - iii. The longer time spent in the air means that the potato will both fly higher and further, as it isn't accelerating towards the ground at such a high rate.

## Torque and Equilibria

1.

a. i.



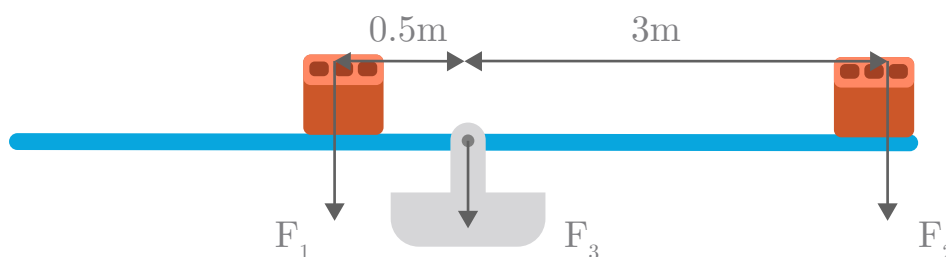
ii. An object is in rotational equilibrium when it has no net torques acting upon it. Since the seesaw is staying level, it is not rotating, which means that it has no net torques and is therefore in rotational equilibrium.

$$\begin{aligned} \text{iii. } \tau_{\text{anticlockwise}} &= Fd \\ \tau_{\text{anticlockwise}} &= mgd \\ \tau_{\text{anticlockwise}} &= 36 \times 9.8 \times 1 = 352.8\text{Nm} \end{aligned}$$

$$\begin{aligned} \text{iv. } \tau_{\text{clockwise}} &= Fd \\ \tau_{\text{clockwise}} &= mgd \\ \tau_{\text{clockwise}} &= 12 \times 9.8 \times 3 = 352.8\text{Nm} \end{aligned}$$

v. We know the seesaw is in rotational equilibrium as it is not rotating. This means that there must be no net torques acting on it – so the clockwise and anticlockwise torques cancel out. This is the result we have obtained assuming the second brick has a mass of 36kg, so it must be true.

b. i.



$$\begin{aligned} \text{ii. } \tau_{\text{anticlockwise}} &= Fd \\ \tau_{\text{anticlockwise}} &= mgd \\ \tau_{\text{anticlockwise}} &= m \times 9.8 \times \frac{1}{2} = 4.9m \end{aligned}$$

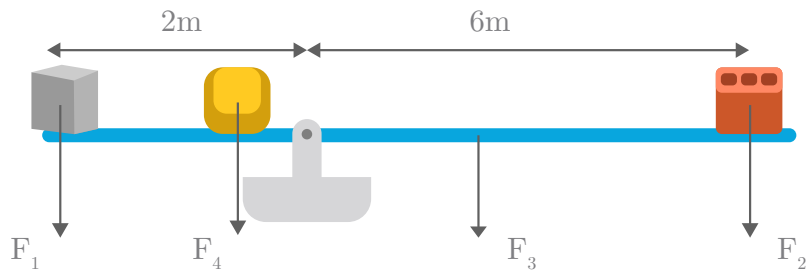
$$\begin{aligned} \text{iii. } \tau_{\text{clockwise}} &= Fd \\ \tau_{\text{clockwise}} &= mgd \\ \tau_{\text{clockwise}} &= 12 \times 9.8 \times 3 = 352.8 \end{aligned}$$

iv. Rotational equilibrium means there is no net torque, so:

$$\begin{aligned} \tau_{\text{anticlockwise}} &= \tau_{\text{clockwise}} \\ 4.9m &= 352.8 \end{aligned}$$

$$\begin{aligned} \text{v. } m &= \frac{352.9}{4.9} \\ m &= 72\text{kg} \end{aligned}$$

c. i.



ii. The weight force of the titanium and gold brick.

iii. The weight force of the regular brick and the seesaw plank.

iv. The centre of mass is at the halfway point of the plank, so  $\frac{6+2}{2} = 4$  from the edges. It is therefore 2m from the pivot.

$$\begin{aligned} \text{v. } \tau_{\text{anticlockwise}} &= \tau_{\text{titanium}} + \tau_{\text{gold}} \\ \tau_{\text{titanium}} &= Fd = mgd = 72 \times 9.8 \times 2 = 1411.2 \\ \tau_{\text{gold}} &= Fd = mgd = 144 \times 9.8 \times 1 = 1411.2 \\ \tau_{\text{anticlockwise}} &= 1411.2 + 1411.2 \end{aligned}$$

$$\begin{aligned} \text{vi. } \tau_{\text{clockwise}} &= \tau_{\text{plank}} + \tau_{\text{brick}} \\ \tau_{\text{plank}} &= Fd = mgd = 100 \times 9.8 \times 2 = 1960 \\ \tau_{\text{brick}} &= Fd = mgd = 12 \times 9.8 \times 6 = 705.6 \\ \tau_{\text{clockwise}} &= 1960 + 705.6 = 2665.6\text{Nm} \end{aligned}$$

vii. Rotational equilibrium means there is no net torque, so:

$$\begin{aligned} \tau_{\text{anticlockwise}} &= \tau_{\text{clockwise}} \\ 1411.2 + 1411.2 &= 2665.6 \end{aligned}$$

$$\begin{aligned} \text{viii. } l &= \frac{2665.6 - 1411.2}{1411.2} \\ l &= 0.889\text{m} \end{aligned}$$

## Springs

1.

- a. i.  $E_p = \frac{1}{2}kx^2$ , so we need the compression (which we have) and the spring constant (which we do not have).

ii.  $F = kx$ . Since the force is the weight force, we can calculate the weight force Lee is exerting on each spring and then find the spring constant.

iii.  $F_g = mg$

$$F_g = 65 \times 9.8 = 637\text{N}$$

Since Lee is spreading his weight across 16 springs, the force exerted on each spring is

$$\frac{637}{16} = 39.81\text{N}$$

iv.  $F = kx$

$$k = \frac{F}{x}$$

$$k = \frac{39.81}{0.038} = 1047.70\text{Nm}^{-1}$$

v.  $E_p = \frac{1}{2}kx^2$

$$E_p = \frac{1}{2} \times 1047.70 \times 0.038^2$$

$$E_p = 0.756\text{J}$$

- b. i. By curling into a ball, Lee is distributing his mass across fewer springs, meaning the force on each spring is increased.

ii.  $F = kx$ , so an increase in force means an increase in compression since the spring constant is unchanged.

iii.  $E_p = \frac{1}{2}kx^2$ , so if the compression in each spring is increased, more energy is stored in them because the spring constant doesn't change.

2.

- a. i.  $F = kx$ , so we are missing  $x$ , the compression of the spring.

ii.  $E_p = \frac{1}{2}kx^2$

$$x = \sqrt{\frac{2E_p}{k}}$$

$$x = \sqrt{\frac{2 \times 0.025}{112}}$$

$$x = 0.0211\text{m}$$

iii.  $F = kx$

$$F = 112 \times 0.0211$$

$$F = 3.26\text{N}$$

$$\text{iv. } F = mg$$

$$m = \frac{F}{g}$$

$$m = \frac{2.36}{9.8}$$

$$m = 0.241\text{kg}$$

**b.** *i.* Energy is conserved, assuming negligible loss as heat and sound due to air resistance.

*ii.* When the ball is held up before the drop it has gravitational potential energy stored in it. As it falls, this converts into kinetic energy, which then becomes elastic potential energy stored in the spring as the ball compresses it. Assuming no loss, the energy stored in the spring is equal to the energy stored in the ball before the drop.

*iii.* The gravitational potential energy that the ball must start with is equal to the elastic potential energy it ends with, so  $E_p = 0.025\text{J}$

$$E_p = mg\Delta h$$

$$\Delta h = \frac{E_p}{gm}$$

$$\Delta h = \frac{0.025}{9.8 \times 0.241}$$

$$\Delta h = 0.0106\text{m}$$



## Work and Energy

1.

- a. i. The ball only has gravitational potential energy at the top of the hill. Since the ball starts from rest, it has no kinetic energy.

ii.  $E_p = mgh$

$$E_p = 2 \times 9.8 \times 3 = 58.8$$

- iii. When the ball is at the top of the hill, all of its energy is stored as potential energy. As it rolls down the hill, this is converted to kinetic energy. At the bottom of the hill, all of its energy would have been converted, so  $E_k = 58.8\text{J}$ .

iv.  $E_k = \frac{1}{2} mv^2$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2 \times 58.8}{2}} = 7.67\text{ms}^{-1}$$

- b. i. When the ball compresses the spring and stops it converts all of its kinetic energy into the potential energy stored in the spring due to conservation of energy. So, the potential energy stored in the spring is  $E_p = 58.8\text{J}$

ii.  $E_p = \frac{1}{2} kx^2$

$$k = \frac{2E_p}{x^2}$$

$$k = \frac{2 \times 58.8}{1^2} = 117.6\text{Nm}^{-1}$$

- c. i. Conservation of energy.

- ii. The elastic potential energy is fully converted to kinetic energy as the ball begins to move back towards the hill.

- iii. The kinetic energy is converted to gravitational potential energy, so the ball will slow down as it gets higher.

- iv. As much as it started with (58.8J), since none has been lost.

- v. It will roll all the way back up to where it was, since no energy is lost.

- d. i. All of the energy was converted from gravitational potential to kinetic to elastic then back again. Since none is lost, the ball ends up with the same gravitational potential energy, so it rolls back to its starting point.

- ii. Changing the mass doesn't affect this conclusion because the energy will always be conserved.

- iii. Changing the mass of the ball doesn't affect how high it rolls up the hill because the energy is conserved.

- e. i. All of the energy was converted from gravitational potential to kinetic to elastic then back again. Since no energy is lost, the ball ends up with the same gravitational potential energy, so ends up rolling back to its starting point.
- ii. Changing the steepness of the hill doesn't affect this conclusion, as there is conservation of energy.
- iii. Changing the steepness of the hill doesn't affect how high it rolls up the hill, as the energy stays the same throughout due to the conservation of energy.
- f. i. Work is the transfer of energy from one object to another, or the conversion of energy from one form to another.
- ii. 58.8J
- iii. 58.8J
- iv. The weight force due to gravity.
- v. The weight force due to gravity.
- g. i.  $P = \frac{W}{t}$
- ii.  $P = \frac{58.8}{5.9}$
- $P = 9.97W$

## Section Three

# Practice Exam

### Question One

- a. Acceleration is the rate of change in velocity, which is a vector quantity that has both size and direction. While the discus' speed is constant, it is constantly changing the direction of its motion, so its velocity is changing, so the discus is accelerating.
- b. The velocity of the discus is always tangential to the circular path. When Lucy lets go of the discus it stops accelerating towards the centre of the circular path and the velocity causes it to continue moving in a straight line in the direction it was already moving before she released the discus.

- c. The tension force in Lucy's arm provides the centripetal force which acts towards the centre of the circular path.

$$v = \frac{d}{t}$$

$$v = \frac{\pi D}{T}$$

$$v = \frac{\pi \times 1.68}{1.2 / 3} = 13.19468915 \text{ms}^{-1}$$

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{2.0 \times 13.19468915^2}{1.68 / 2} = 414.5 \text{N}$$

- d.  $v_{\text{vert}} = 13.2 \times \sin(10) = 2.292155945 \text{ms}^{-1}$

Finding time to reach top of arc:

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{0 - 2.292155945}{-9.8} = 0.23 \text{s}$$

So the discus will take  $0.23 \times 2 = 0.46 \text{s}$  to land.

Now to calculate how far it travelled in that time:

$$v_{\text{horiz}} = 13.2 \times \cos(10) = 13 \text{ms}^{-1}$$

$$d_{\text{horiz}} = v_{\text{horiz}} t$$

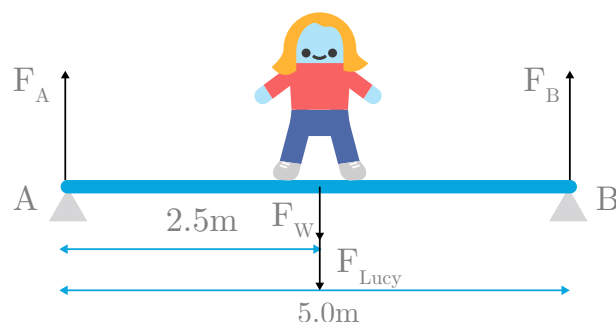
$$d_{\text{horiz}} = 13 \times 0.46 = 5.98 \text{m}$$

- e. Momentum is not conserved as the discus swings because there is an external force acting on it, the centripetal force provided by the tension force.

Momentum is not conserved when the discus is thrown because gravity is an external force acting on it.

## Question Two

a.



b. Spring constant of the board:

$$F = kx$$

$$F_W + F_{Lucy} = kx$$

$$k = \frac{F_W + F_{Lucy}}{x}$$

$$k = \frac{8 \times 9.8 + 58 \times 9.8}{0.065} = 9951 \text{ Nm}^{-1}$$

Elastic potential energy:

$$E_p = \frac{1}{2} kx^2$$

$$E_p = \frac{1}{2} 9951 \times 0.065^2 = 21.02 \text{ J}$$

c. When Lucy is above the plank, she has gravitational potential energy which converts into kinetic energy as she falls. When she lands on the beam her kinetic energy transforms into elastic potential energy stored in the beam.

The higher Lucy falls from, the more gravitational potential energy she has, and so she will have more kinetic energy when she lands on the plank. This means the plank sags more as  $E_p = \frac{1}{2} kx^2$ , where  $k$  is constant.

d. Torque when Lucy is in the centre:

$$\tau_i = Fd$$

$$\tau_i = mgd$$

$$\tau_i = 58 \times 9.8 \times 2.5$$

$$\tau_i = 1421 \text{ Nm}$$

Torque when Lucy is 2m to the left:

$$\tau_f = Fd$$

$$\tau_f = mgd$$

$$\tau_f = 58 \times 9.8 \times (2.5 - 2)$$

$$\tau_f = 284.2 \text{ Nm}$$

The torque around A decreases by  $1421 - 284.2 = 1136.8 \text{ Nm}$  when Lucy moves 2m towards it.

## Question Three

a. Momentum is conserved during this collision.

b. Conservation of momentum implies  $p_{\text{before}} = p_{\text{after}}$ . Since the 8-ball is initially stationary,

$$p_{\text{before}} = m_{\text{cue}} v_{\text{cue}} + m_{8\text{-ball}} \times 0$$

$$p_{\text{before}} = 0.17 \times 12 = 2.04$$

$$\text{Therefore, } p_{\text{after}} = 2.04 \text{ kgms}^{-1}$$

After the collision, the balls have a total combined mass of  $m_{\text{cue}} + m_{8\text{-ball}}$ , and a velocity of  $v_{\text{joined}}$ .

$$p_{\text{after}} = (m_{\text{cue}} + m_{8\text{-ball}}) v_{\text{joined}}$$

$$2.04 = (0.170 + m_{8\text{-ball}}) \times 7$$

$$\frac{2.04}{7} = 0.170 + m_{8\text{-ball}}$$

$$m_{8\text{-ball}} = \frac{2.04}{7} - 0.170$$

$$m_{8\text{-ball}} = 0.121 \text{ kg}$$

So, the mass of the 8-ball is 0.121 kg.

c. We have:  $v_i = 7.0 \text{ ms}^{-1}$ ,  $v_f = 0 \text{ ms}^{-1}$ ,  $F = -9.0 \text{ N}$

We want:  $t$

Remember that  $F = ma$

$$a = \frac{F}{m} = \frac{-9}{0.121 + 0.170} = \frac{-9}{0.291} = -30.93 \text{ ms}^{-2}$$

Now to find  $t$  we can use  $v_f = v_i + at$

$$0 = 7 + -30.93 \times t$$

$$30.93 \times t = 7$$

$$t = \frac{7}{30.93}$$

$$t = 0.226 \text{ s}$$

d. To calculate both forces, we can use  $\Delta p = F\Delta t$ , with  $\Delta t = 0.2 \text{ s}$

$$\Delta p_{\text{cue}} = p_f - p_i = 12 \times 0.17 - 7 \times 0.17$$

$$\Delta p_{\text{cue}} = 0.85 \text{ kgms}^{-1}$$

$$F_{\text{cue}} = \frac{\Delta p_{\text{cue}}}{\Delta t} = \frac{0.85}{0.2}$$

$$F_{\text{cue}} = 4.25 \text{ N}$$

$$\Delta p_{8\text{-ball}} = p_f - p_i = 0 \times 0.121 - 7 \times 0.121$$

$$\Delta p_{8\text{-ball}} = -0.85 \text{ kgms}^{-1}$$

$$\Delta p_{8\text{-ball}} = \frac{\Delta p_{\text{cue}}}{\Delta t} = -\frac{0.85}{0.2}$$

$$\Delta p_{8\text{-ball}} = -4.25 \text{ N}$$

These two forces have equal magnitude but act in opposite directions – this is Newton's second law; every action has an equal and opposite reaction. The force on the cue ball from the 8-ball is equal and opposite to the force on the 8-ball due to the cue ball.

- e. We have:  $v_i = 7.0\text{ms}^{-1}$ ,  $m = 0.29\text{kg}$ ,  $d = 0.5\text{m}$ ,  $a = -30.882\text{ms}^{-2}$

We want:  $v_f$

Using  $v_f^2 = v_i^2 + 2ad$ ,

$$v_f^2 = 7^2 + 2 \times (-30.93) \times 0.5 = 49 - 30.93 = 18.07$$

$$v_f = \sqrt{18.07} = 4.25\text{ms}^{-1}$$

Now we can calculate the kinetic energy,

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.29 \times 18.07$$

$$E_k = 2.62\text{J}$$

From conservation of energy,  $E_p = 2.62\text{J}$  also

$$E_p = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{2E_p}{k}}$$

$$x = \sqrt{\frac{2 \times 2.62}{12}}$$

$$x = 0.66\text{m}$$

We assumed that no energy is lost to friction, heat, etc. so that it was perfectly conserved.

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