L3 DIFFERENTIATION
CHEAT SHEET

STUDY TIME

EXAM STRUCTURE

- This exam paper contains three big questions, under each question, there are around 5 small questions with the excellence question being the last one. You must answer the excellence questions to get an excellence grade.
- First two questions are usually at an achieve level. These questions are very straightforward and require you to use the basic differentiation skills to solve.
- Merit and excellence questions require you to figure out what the examiners are testing on by transferring words into mathematical equations.
- This paper involves a mixture of calculation-based questions and graph analysing questions. Questions involved looking at graph are also straightforward and you can realize what skills they are testing directly.

HOW TO START A QUESTION

- If you are given an equation which you cannot solve directly, thinking about rewriting it in another form.
  - For example, $y = \sqrt{a}$ is hard to differentiate, but by writing it as $y = a^{½}$, it can be differentiated easily.
- For questions involving a lot of words explanation, start by reading each line and use mathematical equations to represent any information they give to you.
  - For example, 'line A is normal to the parabola when x = 3', if we use B to represents the line tangent to the parabola when x = 3, this piece of information can be rewritten as gradient(A) × gradient(B) = −1. This helps you to write equations or simultaneous equations later and this skill is very important for excellence questions.
- If the question is talking about the volume of a cone or the area of a box, draw the graph and label all information on it to help you understand.

CLASSIC QUESTIONS

- Direct differentiation
  - These are the basic skills that you should be very confident about. This involves differentiating log equations, exponential equations at so on. Familiar yourself with the formula sheet can help you find the correct method to use under exam conditions.
- Find the equation of a straight line
  - There are many ways of asking this question but they require the same skill to solve it.
  - For example, 'find the equation of the line that is tangent to the parabola $y = x^2$ at x = 3', 'find the gradient of the line that is tangent to the parabola at x = 3', 'find where a line with a gradient of 5 intersects the parabola if the line is normal to the curve'.

- All these questions require you to do some differentiation of the original curve. **Remember that by integrating, you get an equation representing gradients of all lines that are tangent to the curve.**
- To find the equation of a line, use the equation **y = ax + b**. You need either one point and the gradient or two points to find the equation of a line. So clearly written down everything you know and see if you have enough information.

- Analysing the graph
  - This question requires you to look at the graph to find out the region where the graph is increasing, decreasing, turning, etc. Below are the common ways for examiners to ask this question:
  - Write the x value when the graph is increasing/ decreasing.
  - Write the x value which gives the graph a turning point.
  - Write the x value when the graph concaves down/up.
  - Is the graph continuous/discontinuous when x = 2?
  - Write all x values when the graph is not differentiable.
  - These questions are asked directly and the skills that are being tested are very obvious. You are required to get most of it correct to get a merit.
- Asking for the maximum/minimum of a variable
  - For example, you are making a six-sided cube using a square paper which is 50cm × 50 cm. What is the length of the cube that can give its maximum volume?
  - These questions always contain a lot of words, so it is important for you to draw the graph and use mathematical equations to represent the information.
  - Once you see words like 'maximum', 'minimum', the steps that you need to follow is **writing the equation, differentiation, making it equals zero and solve.**
  - To find whether it is a maximum or minimum value, differentiate the original equation twice and compare the value with zero. **Always remember to write your comparison down before you come up with a conclusion.**
- Asking for the rate of change
  - For example, the volume of the cone is increasing at a rate of 3 cm^3/min, the cone has a radius of 3cm and a maximum height of 10cm. Find the rate of change of the height of the cone.
  - Again, it is important to **write mathematical equations to represent the information**. In this question, the volume of the cone is increasing at a rate of 3 cm^3/min, so, $\frac{dV}{dt} = 3$.
  - When you see words like 'the rate of', using **chain rule** can help you develop an equation and find out what you need to work on. In this question, $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$, $\frac{dh}{dt}$ is the thing you want to know, $\frac{dV}{dt}$ is given, so the only thing you need to work out is dhdV. Using chain rule help you stay on the right track so always using it when you solve this kind of question.

WHAT IF YOU'RE STILL NOT CONFIDENT ABOUT HOW TO DO A QUESTION?

- Practice makes perfect. But you don't have to answer all excellence questions to get an excellence. Starting from the achieve question and finishing all merit questions, then have a look at all three excellence questions and choose the one you feel most confident with. Four merit questions and one excellence question can give you an excellence grade as well.

HOW TO PREPARE

- Calculus is all about practice but it does not mean you should force yourself to do hundreds or thousands of questions to prepare for it. Making conclusions about your mistakes is more important than keep trying new questions and making the same mistakes each time. You need to analyse your mistakes and definitely avoid making it next time.
- Doing past papers are helpful. Try at least three past papers under exam conditions before your test and mark it yourself. Marking papers help you understand what the examiners are looking for and also help you realize why you getting something wrong.
- NZQA does not provide many past papers for you to do, so once you finish one paper but do not feel so good about it, leaving it for 2 or 3 days and do it again. When you do it again after some days, it will become almost a new paper for you and you can use this chance to see if you have successfully avoided making the same mistakes.
- Last, check the Study Time websites for notes and checklists that can help you get a more solid understanding of the content.