L3 INTEGRATION
CHEAT SHEET

STUDY TIME

EXAM STRUCTURE

- This cheatsheet will give you an idea of when to use each method of integration, and what you might have to do to integrate the problem.

POLYNOMIALS AND RATIONAL EXPRESSIONS

- In general, whenever you see a variable on the denominator of a fraction or a radical, **rewrite the expression with a negative power and with fractional indices**, an example of this could be:
  - $\int (\frac{3}{x^2} + 2\sqrt{x})dx = \int (3x^{-2} + 2x^{½})\,dx$

  Now it's much easier to see how the polynomial rule of integration applies.
- There are a few different ways that a rational expression could be simplified but I'll try give an idea of a basic one that comes up all the time, if given a single term in the denominator then you will *decompose* the fraction using the rule $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ like so:
  - $\int \left(\frac{4x^2 - 2\sqrt{x}}{2x}\right)dx = \int \left(\frac{4x^2}{2x} - \frac{2x^{½}}{2x}\right)dx = \int \left(2x - x^{-½}\right)dx$

  Which can again be easily integrated using the polynomial rule. Don't forget the + C if it's indefinite!
- A mistake that I often see is when a problem looks like it will need to be simplified using something like this but there is a linear factor on the bottom in the form $(x - a)$ like:
  - $\int \left(\frac{3}{x-2}\right)dx$

  But this is actually an integral that results in the natural logarithm, besides, this can't be decomposed at all!

REVERSE CHAIN RULE

- The reverse chain rule is used for *composite functions* where we normally wouldn't be able to integrate, in general we are looking at an expression where there is a function and its derivative in the expression, an example of this could be:
  - $\int 2x \cdot \sin(x^2)\,dx$
- The goal is to get the whole thing in terms of something else, we will call this u. We also need to get rid of the dx, we can do this by taking $u = x^2$ and so $du = 2xdx$. The integral becomes:
  - $\int \sin(u)\,du$

  Which is easy to integrate!
- You should get very comfortable with this idea, it's used all the time in calculus, a good idea would be to try

come up with some examples, sometimes we will have to manipulate our substitution to make it work. In general, these will look like:

- $\int f'(x) \cdot f(x)\, dx$

- Another example of when to use *u-subsitution* is when a polynomial is taken to a power like:
  - $\int (4x + 3)^7\, dx$

You could try to expand this but it would be really difficult, the alternative is to substitute as we normally would, don't forget to replace the dx with du though!

AREAS UNDER CURVES AND BETWEEN FUNCTIONS

- Remember: **Area under a curve is signed**, what does this mean? Well, if the area is under the x-axis then it is *negative area* and if it's above the x-axis it's *positive area*. Use this idea to explain why the average area of a sine or cosine graph is 0 to wrap your head around this idea.
- One useful application of integration is that it allows us to find the area under a curve, a definite integral will find the area between two points, a common question in NCEA is that you are given an area and an integral with an unknown constant, asking you to find the constant. Here you set the definite integral equal to the area and do the integration as you normally would but solve for the unknown constant.
  - $\int_a^b f(x)\, dx = \textit{Area under } f(x) \textit{ between } a, b$
- Another common area question is to the find the *area between two curves*, the trick here is that you find the definite integral of the top curve subtracted by the bottom curve but **careful** because which curve is on top can change, so we also need to find **the intersections** and then sum a couple of integrals together. In general:
  - Area $= \int_a^b \textit{Top Curve - Bottom Curve}$
- Finding intersections is relatively easy, just set one function equal to the other and solve for the x values, these are now your bounds. Then just add the integrals between those points, being careful to integrate the **top function – bottom function** to find the area.