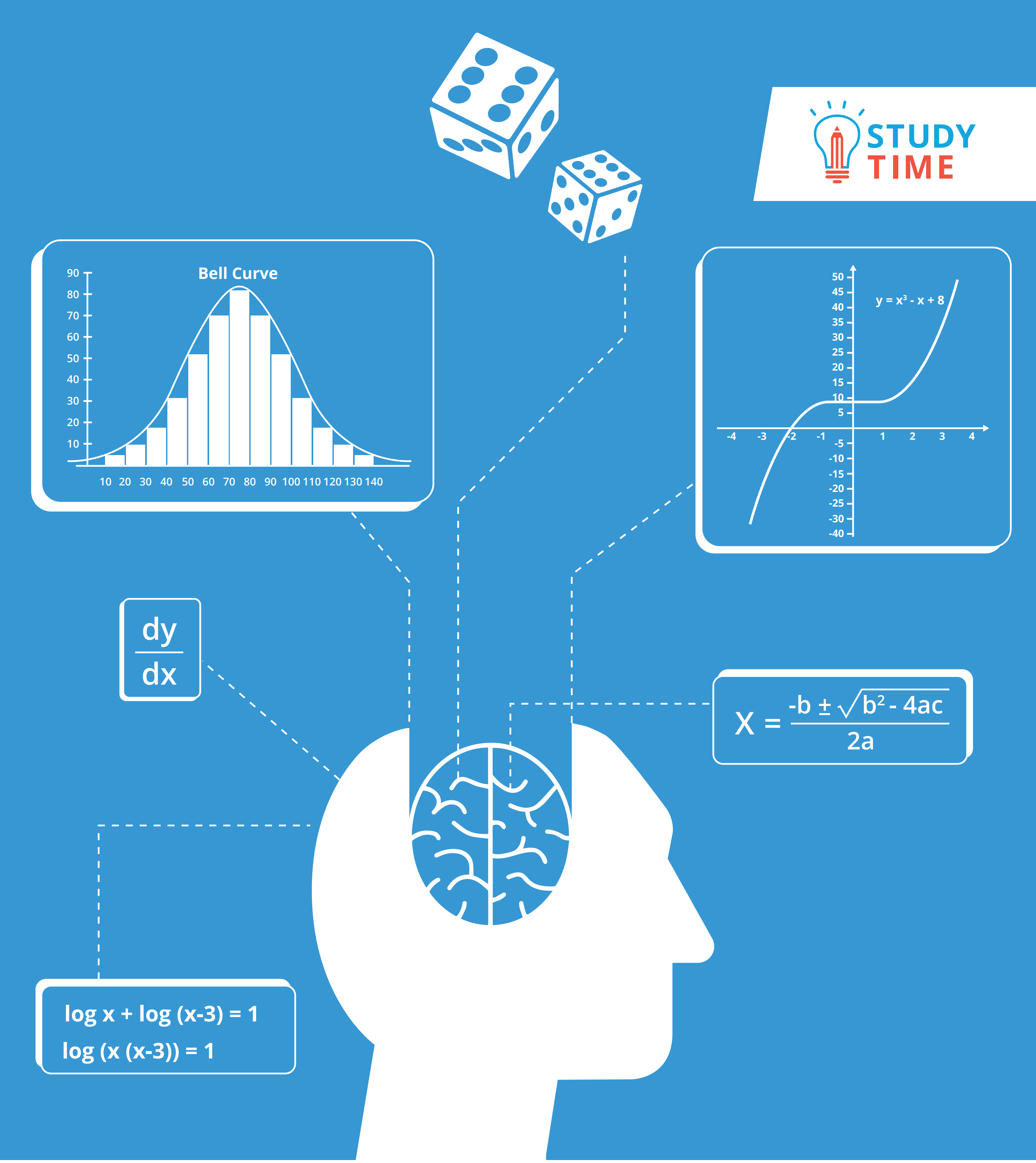


CALCULUS

MATHS

Level 2

NCEA | Walkthrough Guide

INTRODUCTION

Calculus is, hands-down, the most awesome kind of maths you'll do this year – and sure, maybe we're a little biased, but it's *super* awesome because you can see it happening in the *real world*.

What do we mean? Well, a lot of calculus is about measuring *rates*. That's rates as in how *much* something changes. And if you've ever spent at least one second living and breathing, you'll know that pretty much everything is changing *all the time*.

With calculus, we can write equations to model how fast the population of your hometown is growing. We can calculate the angle of the sun's rays to a solar panel. We can do all sorts of weird and wonderful things – bet you're glad you picked up this walkthrough guide now!

What we'll be covering in this walkthrough guide

Calculus always kicks off with *gradients*, which thankfully, is a word you learnt last year and the year before. Then we'll teach you *differentiation*, which is basically a clever way of finding the gradient on a graph. You'll learn what a *function* is, and even how to *anti*-differentiate.

Next, graphs – woohoo! Pretty pictures! You'll learn how to draw the derivatives of graphs and then draw graphs from the derivatives.

Finally, the fun stuff! Kinematics, which is all about how acceleration,velocity, distance and time work together.

A word on exam strategy

Your calculator can be quite essential for this assessment, BUT we don't recommend you learn *everything* on it. It's better for you to know how to do most things *by hand,* because then you'll have a better understanding of how it works.

And showing your working is a *must*, so you have to know how to do the working in the first place. A l w a y s show your working! If, in some crazy twist of fate, you get the answer *wrong*, you can always redeem yourself if you were on the right track with your working!

Finally, *check your answers*. Calculus isn't a wordy exam, so your grade pretty much rides on calculations. Like we said, you'll still get good marks if your *working* was right, but why not go the whole hog and get the answer right as well? All it takes is a quick once-over with your Casio, and BOOM! EXCELLENCE!

We understand that this is a new maths topic, so, that's why, at StudyTime, we're pretty much GCs (good citizens), so to help you out, we've made this guide in plain English as much as we can. We've also included a glossary for some of the key terms that you'll need to master for your exam.

If learning key words first off scares you (or bores you), then focus on understanding the concepts the first time around, and then memorise the definitions.
In fact, in this guide, we focus on helping you to understand the concepts first. We use examples and analogies to help you understand maths in a way that is fun, and makes sense in the real world.

However, the language we use isn't always something you can directly write in your exam! When this is the case, we offer a more scientific definition or explanation (in a handy blue box) underneath! These boxes are trickier to understand on your first read through, but contain language you are allowed to write in your exam! Look out for them to make sure you stay on target!

GRADIENTS AND DIFFERENTIATION

Calculus is an interesting topic in Level 2 maths because, unlike probability and algebra, there's likely to be a lot of topics that you've never heard of before. In fact, most students starting out Level 2 don't actually know what 'calculus' even means – let alone terms like 'differentiation' and 'integration'.

If this is you, don't worry! We're going to start this topic with a term you are hopefully familiar with – the 'gradient'. In fact, although it may seem overwhelming at times, all this entire topic really comes back to is exploring the gradient.

Hopefully you remember linear graphs and gradients from previous years. But if not, we'll give you a quick rundown before we crack into the new stuff. In fact, here's a handy list of what we'll be covering in this section:

- What gradients are and how they change
- WHAT YOU'VE ALL BEEN WAITING FOR – Differentiation!
- Finding the gradient at a certain point
- Anti-differentiation…otherwise known as integration

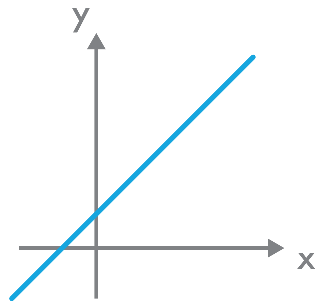
Gradients

The *gradient* is the steepness of a line. In other words, this means how 'much' the line is moving up, compared to how fast it is moving across. Put in technical maths language, this really means:

The *rate* that y is changing compared to the rate that x is changing.

The gradient measures how one variable changes in relation to another.

When we say 'the rate' we mean 'how much'. Let's see this in action. You remember line graphs, right? Something like this:



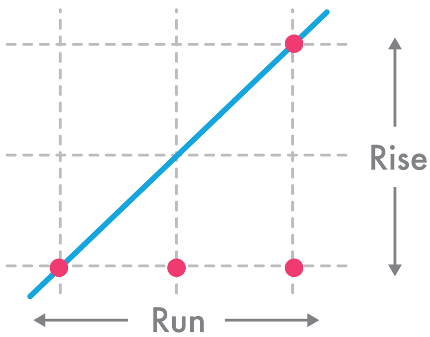
And it has *this* general equation:

$$y = mx + c$$

Where *c* is the y-intercept and *m* is the *gradient*.

The gradients of straight lines are *constant*. That means they *never change*.

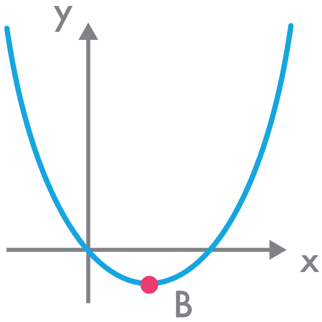
You can find the gradient using *rise/run*. Because the gradient is the same at any point on the line, you can do this calculation with any two points on the graph.



But don't stop there – you remember parabolas, don't you? They have this general equation (where a, b and c represent any number):

$$y = ax^2 + bx + c$$

...and they look like this:



Now that we know what they look like, let's try to work out the gradient of one!

You will hopefully remember that the gradient of a linear graph is found using the formula:

$$\frac{rise}{run}$$

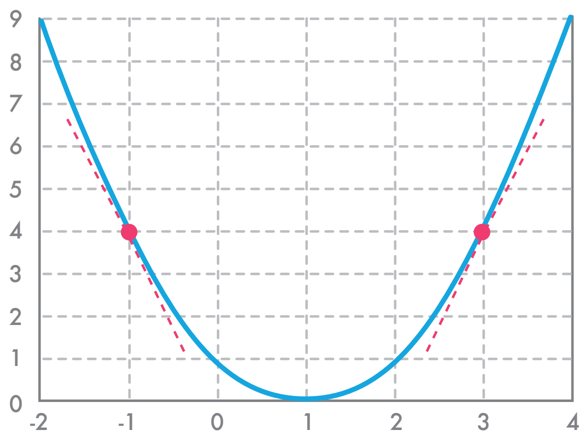
Where you count up how far the gradient moves up from a certain point, and divide it by how far it moves across.

However, if you try to use this method on a non-linear graph (like a parabola), you are going to run into trouble! This is because, on a parabola, the gradient changes!

In fact, there are actually no two points where the gradient is the same.

Therefore, instead of calculating the gradient of the whole graph, when we look at non-linear graphs, we actually want to find the gradient at a specific point!

On a parabola, the gradient at a specific point is found by drawing a straight line at a tangent to the graph at that point. In case you don't remember from last year, a tangent is simply a line that touches the graph exactly once. The gradient of this line is how we describe the gradient of the parabola at each point:



STOP AND CHECK:

Cover up the book! Now answer these questions in your own words:

- What's the gradient?
- Does a linear line have a constant or changing gradient?

Differentiation!

Differentiation is a tongue-twisting word that actually just means *finding the gradient* of a graph. That's right – you thought we were going to be teaching you some mathematical magic, but this entire topic is just about finding gradients.

In the last section, we left you a bit of a cliff-hanger. We told you that the gradient of a parabola is always changing – but we didn't tell you how to find it exactly.

This is where differentiation comes in!

Because the gradient is constantly changing, we don't find a fixed value for the gradient (like on a straight line graph). Instead, we work out an equation which we can use to find the gradient at any point.

If you have, say, a quadratic equation (which you can find from any parabola), you simply *differentiate* the original equation to find the equation of its *gradient*!

Differentiation is used to find the derivative of a function. The derivative tells us the gradient at any point on its related graph.

When we differentiate quadratic equations, we first rename the y in the equation $f(x)$. This term means *'the function of x'*. 'Function' is just a fancy word for 'equation', so this really just means *'the equation about x'.*

A function is an equation which relates multiple variables together.

We can then call the *gradient function* of $f(x)$ any of these three things:

$$f'(x) \qquad \frac{dy}{dx} \qquad y'$$

Differentiating an equation is all about converting a function into its gradient function. Luckily, there's actually a simple process we can use to do this.

The simple process looks like this:

1) *If* $f(x) = x^n$*, then* $f'(x) = nx^{n-1}$

Let's break that down a bit:

- The terms tell us that we are going from a 'function of x', or *f(x)*, to its gradient function, or *f'(x). (For reference, the f'(x) is pronounced as 'f prime of x')*
- The 'n' value tells us that any time we see an exponent (or power) in the original function, we put it in front of the 'x' symbol (making it a co-efficient) and subtract 1 from the exponent

A co-efficient is a number that appears before a variable. In differentiation, any power is converted to a co-efficient.

Alright. Let's look at an example.

If $f(x) = x^6$*, then* $f'(x) = 6x^5$

See what we did? To differentiate, we *multiplied at the front by the power and reduced the power by one.*

2) Now, if we had a number in front of x, what would happen? It'd get *multiplied* by the power, wouldn't it?

If $f(x) = 2x^6$*, then* $f'(x) = 12x^5$

See what we did there? We multiplied the original co-efficient by the exponent (2 x 6 = 12).

3) If there's no power, the x simply drops off and leaves the number.

If $f(x) = 12x$*, then* $f'(x) = 12$

This is because an x value with no visible power, technically has a power of 1, so subtracting 1 from 1 gives us zero. Anything to the power of zero is one, so we are technically just multiplying the number by 1 instead of x.

4) If there's no *x* at all, then the derivative of that number is just *zero*.

If $f(x) = 12$*, then* $f'(x) = 0$

5) Your functions aren't going to be just one term. They're going to be several strung together, so you have to differentiate them one-by one.

$$\text{If } f(x) = x^3 + 2x^2 - 3x + 5$$

$$\text{then } f'(x) = 3x^2 + 4x - 3$$

Take some time to look at each individual term in the equations above – and make sure you understand how each was differentiated.

6) Sometimes it's necessary to expand an expression first, or simplify using division.

$$\text{If } f(x) = \frac{4x^2}{2x} = 2x$$

$$\text{then } f'(x) = 2$$

That's a lot of information to take in at once! However, once you get some practice in, you'll realise that differentiation itself is pretty repetitive – and there are only a few different cases to remember.

Make sure to do some practice, and understand everything going on in each of these examples before moving on to the next section!

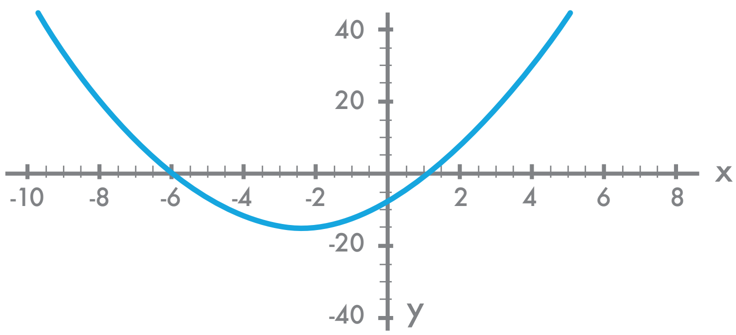
STOP AND CHECK:

Differentiate these functions:

- $f(x) = (2x + 1)(x + 3)$
- What does f(x) mean?

Finding the gradient at a certain point

Differentiation is nice, but what good is it? Well, we can use it to find the *gradient* at a certain point on a line. Say we had this parabola:



Equation of the graph is: $y = x^2 + 5x - 6$

What's the gradient when at the point *(2, 8)*?

First, differentiate the original parabolic equation.

That's what differentiating is – finding the gradient equation! So…

$$f(x) = x^2 + 5x - 6$$

$$f'(x) = 2x + 5$$

Remember that $f'(x)$ is the value of the gradient function at any point on the graph. When we substitute the x value from our coordinates in the gradient function we will have the gradient of our graph at that x value.

Substitute in the *x* value.

Notice we don't use the *y* value. If you're trying to find the gradient, *x* is what you're after!

$$f'(x) = 2(2) + 5$$

Solve to find the gradient.

$$f'(x) = 9$$

The gradient is 9 at the point (2,8).

The key is understanding that when you *differentiate* a function, you're finding the equation of the gradient function. This'll come in handy later on when we get to do fun stuff like kinematics and rates of change.

Onwards!

STOP AND CHECK:

- Find the gradient at the point (2, 1) on the function $y = 2x^2 + 4x + 1$
- When you differentiate, what are you finding?
- What does *f'(x)* mean?

Anti-differentiation, Otherwise Known as Integration

That's the basics of differentiation complete! Now, it's time for our next big calculus concept – integration!

Luckily, integration is just the reverse of differentiation! Instead of beginning with a function for a graph and finding the gradient function, we go from the gradient function to the original function.

In order to show that we are trying to find our original graph equation, some books use the wiggly integral sign, $\int$, to say we're busy integrating. You don't need to use this in your working at level two, but you may see it around, and it's a good habit to build now! To integrate, you just do all of the opposite steps to differentiation:

1) **Increase** the power by one (because we **decrease** by one when we differentiate).
2) **Divide** the co-efficent by the new power (because we **multiply** it when we differentiate).
3) Add a constant, c, on the end. To illustrate why we need the 'c', think about how you would differentiate this equation:

$$f(x) = 2x + 3$$

$$\text{differentiates to: } f'(x) = 2$$

Where does the 3 go? In differentiation, we remove any term which isn't attached to our variable in question. However, when we go backwards during integration, we need a way to bring this number back! That's what the c represents.

Let's get back on track, and see all of those steps in action! Take a look at this question:

$$\int 10x^5 dx$$

We always write *dx* on the end to say, 'this function is in terms of x'. You'll see this notation alongside the wiggly sign.

Start by increasing the power by 1:

$$\int 10x^6 dx$$

Then, divide the whole term by the new power, and add c.

$$\frac{10x^6}{6} + c$$

Of course, it's always nice to simplify down as much as possible. *10* and *6* are both divisible by 2, so let's simplify them.

$$\frac{5x^6}{3} + c$$

Lovely! It pays to remember all your basic algebra skills such as simplifying and rearranging. Now for *c*.

What is c? It's an unknown constant. We know there should be a number there, we just don't know what it is. In most questions, where they don't give you any extra information, you can get away with just writing *c*. That's expected. BUT,

> If the constant (c) CAN be determined, then you will need to find it. This is done using coordinates which you will be given. Let's look how to do this:

Integrate $f'(x) = 4x^3 - 0.5x$ given that the line passes through the point (1, 3.5).

You'd first go through the regular process of integration and get this:

$$f(x) = \int 4x^3 - 0.5x \, dx$$
$$f(x) = x^4 - 0.25x^2 + c$$

And because you KNOW the line passes through point *(1, 3.5)*, you could:

> Substitute in your x and y coordinates to find c.

$$3.5 = (1)^4 - 0.25(1)^2 + c$$

Of course, this takes a bit of rearranging.

$$3.5 = 0.75 + c$$
$$3.5 - 0.75 = c$$
$$2.75 = c$$
$$c = 2.75$$

Now we can chuck that back in and get our fully integrated equation!

$$f(x) = \quad x^4 - 0.25x^2 + 2.75$$

Now that you've got a handle on differentiation and integration, keep reminding yourself how they work together. Remember that differentiation takes you from the function its gradient function, and integration does the opposite!

STOP AND CHECK:

- Integrate $\int 2x^3 - 2\,dx$ given that it passes through point (2, 7).

Quick Questions:

- Integrate $\int (4x + 2x^2)dx$
- Find the gradient of the function $f(x) = 5x^3 + 2x^2 - 4x - 8$ where $x = 3$

FUNCTIONS AND GRAPHS

So far, we've been using a lot of numbers – but we haven't drawn much, or looked at too many graphs! This is a little odd – considering functions and gradients are all about explaining how graphs look!

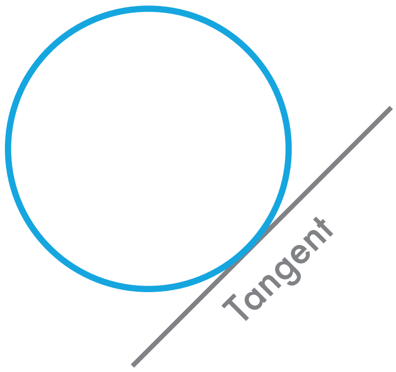
Don't worry, the time has finally come! In this section, we'll be looking at how functions relate to graphs, making sure to cover:

- Tangents to curves
- Calculating the turning points
- Sketching gradient functions

Tangents to Curves

If you remember coordinate geometry, this is going to be nice and familiar.

A tangent is a line that touches the curve at a point, but doesn't intersect it.



Some questions will ask you to find the equation of the tangent to a curve, and thanks to calculus and coordinate geometry, we can do just that in only *three simple steps!*

Find the equation of the line which is tangent to the curve $f(x) = 2x^3 - 7x^2 + 4x - 1$ *at* $(0, -1)$

1) Differentiate to find $f'(x)$. This gives us the equation for the gradient of the tangent.

$$f'(x) = 6x^2 - 14x + 4$$

2) Substitute your given value for x into the derivative to find the gradient at that specific point, *m*.

$$f'(x) = 6x^2 - 14x + 4$$
When $x = 0$:
$$f'(x) = 6(0)^2 - 14(0) + 4$$
$$f'(x) = 4$$

3) Substitute all your values into the equation $y - y_1 = m(x - x_1)$ to find the equation for the tangent.

$$y - y_1 = m(x - x_1)$$

$$y + 1 = 4(x - 0)$$

$$y = 4x - 1$$

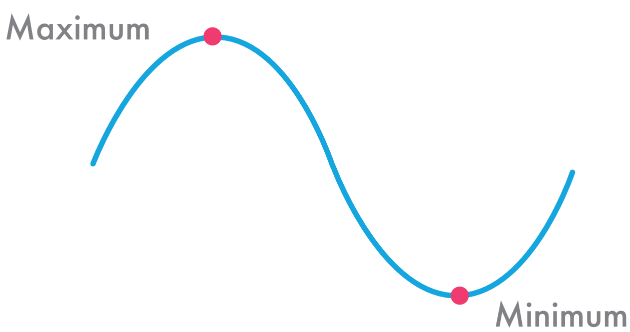
And there you go! It's just a matter of calculating the gradient at the specific point, and substituting into another general equation.

STOP AND CHECK:

- Find the equation of the tangent to the curve $f(x) = -2x^2 + 6x - 1$ at $x = 2$
- What is a tangent?

Calculating the Turning Points

There's two kinds of turning points on a graph – a *maximum* and a *minimum*.

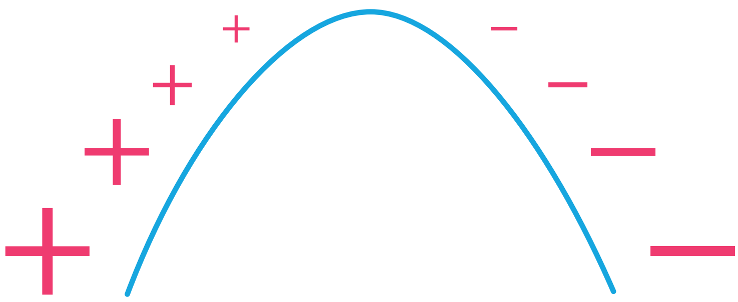


They're called turning points because the graph does a complete u-turn and heads in the other direction, and for a split-second, they're completely flat. That means…

At all turning points, the gradient $f'(x) = 0$.

A turning point is any area on a graph where the graph switches from positive to negative values (or vice versa). Because there is a moment where the gradient is between positive and negative, the gradient at a turning point is always zero.

Turning points are also super easy to find because they're where the gradient changes from *increasing* to *decreasing*. A complete u-turn, like we just said. Take a look:



The line slopes *up up up* increasing – TURNING POINT! – then it slopes *down down down* decreasing! So what kind of turning point does that make? A *maximum*!

When the function is increasing or decreasing

Often, it can be useful to know whether a function is increasing or decreasing. However, you are not always lucky enough to get to see a graph to tell you this.

An increasing section of a function has a positive gradient for that section

Luckily, there is a way to work out what a graph is up to at a specific point using just the equation – and our handy tools of differentiation!

Take this question for example!

Find the turning point of $y = x^2 + 2x$ and identify whether it is a maximum or a minimum

Finding the turning point.

If you've got the turning point, then you know the point where the gradient changes. How do we find the turning point?

- Differentiate
- Equate to zero
- Solve for x

So let's differentiate $y = x^2 + 2x$.

$$\frac{dy}{dx} = 2x + 2$$

Now we equate to zero, because we know that the gradient is zero at turning points.

$$0 = 2x + 2$$
$$-2x = 2$$
$$-x = 1$$
$$x = -1 \text{ when } f'(x) = 0$$

Although we know that the turning point is at 0 when x is -1, we don't know if this is a minimum or a maximum point. Therefore, the next step is to work this out. The easiest way to work this out is to substitute in an x value to see which side of $x = -1$ has a positive gradient and which side has a negative gradient.

Because it is close to the $x = -1$ turning point, let's put $x = 0$ into the gradient equation to see if the gradient comes out as positive.

$$f'(x) = 2x + 2$$

$$f'(x) = 2(0) + 2$$

$$f'(x) = 2$$

Would you look at that – it is positive! We know that −1 is the minimum turning point –as once you get past it, the graph is increasing! But let's check the otherside just to be sure… do this by substituting in −2 as the x value and retrieving the negative f'(x).

STOP AND CHECK:

Why is the gradient zero at a turning point?

Sketching Gradient Functions

We've talked a lot so far about gradient functions – and how to find them using differentiation. If you've forgotten what we've been covering, just remember that the gradient function shows how the gradient of a graph changes over time.

One of the cool things about gradient functions, is that, because they are an equation themselves – we can actually draw them on the same graph as the original function they come from.

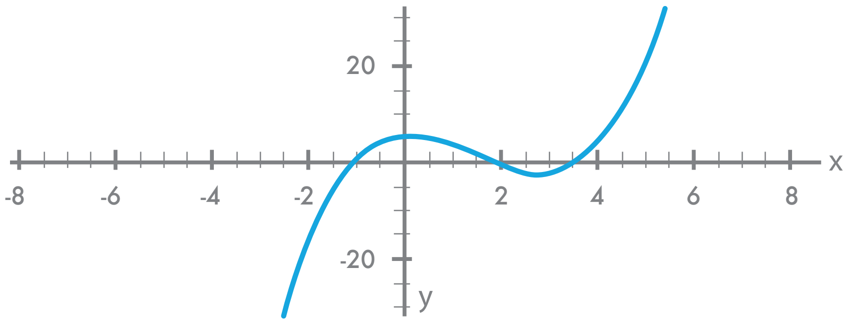
A gradient function creates a graph which describes how the gradient of another graph changes.

In fact, this is something NCEA will often ask you to do!

Of course, the marker doesn't expect you to be Da Vinci-type perfect with your lines – just so long as you get the key points, you're sweet. You're going get some nasty equations to translate – like this:

$$\textit{sketch the gradient function of } y = \frac{1}{2}x^3 - 2x^2 + 3$$

They'll probably be nice and give you the original graph for the function:

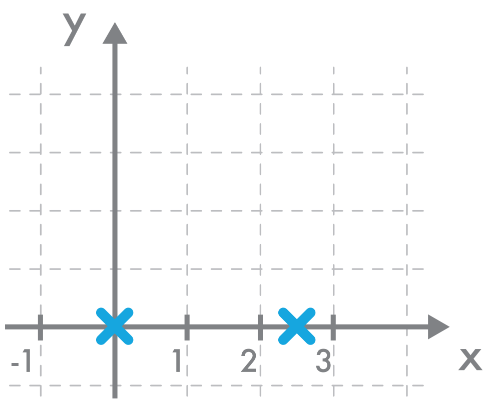


But how do you draw the gradient function?

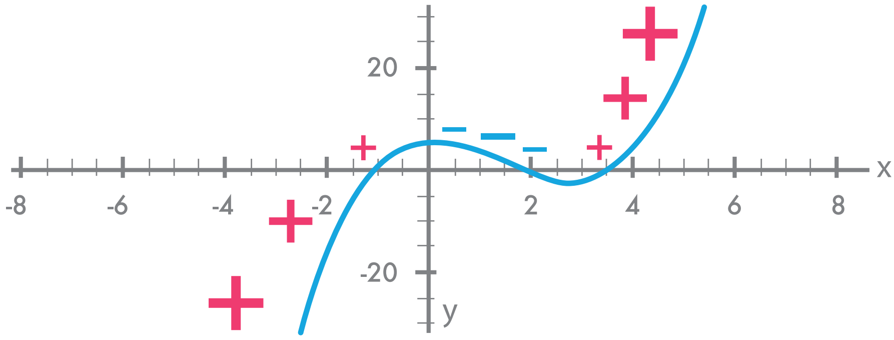
First, find where the function has turning points, then mark where these are on the x axis (in this case the turning points are at '0' and '2').

Turning points always intercept the x-axis on *f'(x)*'s graph.

This is because, at turning points, the gradient, *f'(x)*, equals zero. So we know the y value of the gradient function will be zero at these points.

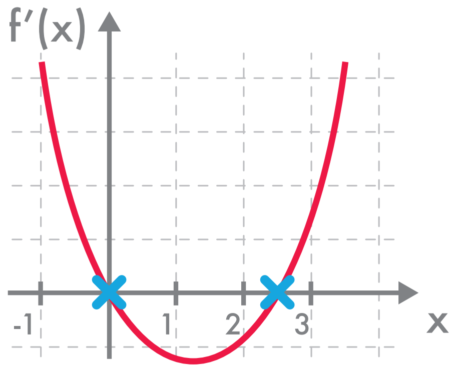


In order to draw the rest of the graph, all you have to do is look at the original graph and see where the line is increasing or decreasing.



It goes *up down up*, so that means the gradient is *positive negative positive*. In other words, we know we must draw the gradient *above* the x-axis, below the x-axis, and *above* it again. Don't worry too much about the shape of the graph. The key things to get correct are where the graph crosses the x axis, and the positive/negative/positive order.

In this case, it looks like:



Oh hey! We've drawn a parabola! Turns out the equation for the gradient of a cubic graph works out to be a parabola.

A cubic graph is a graph with two turning points (one maximum and one minimum). Its gradient function can be drawn as a parabola.

Let's break down why this is by taking our most simple cubic equation. What happens if we differentiate $f(x) = x^3$?

We get $f'(x) = 3x^2$, which is a parabolic equation!

Guess what the gradient of a parabola is? A line! Here's something that will help you remember roughly what your drawing *should* look like:

- *Cubics* always become *parabolas*. This is because cubic graphs have an x^3 term, which is differentiated to get an x^2 term.
- *Parabolas* always become *straight lines*. This is because parabolic graphs have an x^2 term, which is differentiated to get an x term.
- Straight lines always become *horizontal lines*.

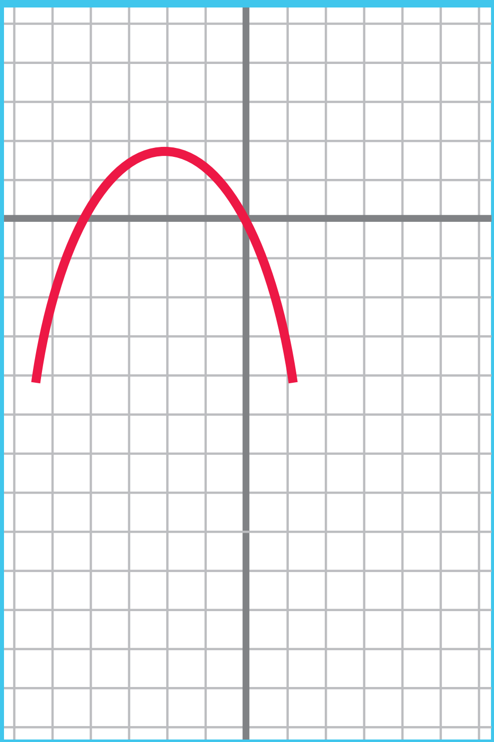
But the same process of:

- Drawing a point (or points) on the x-axis at each turning point.
- Drawing a line (or curve) that goes in the same positive/negative direction as the original graph.

This works for each type of gradient function!

STOP AND CHECK:

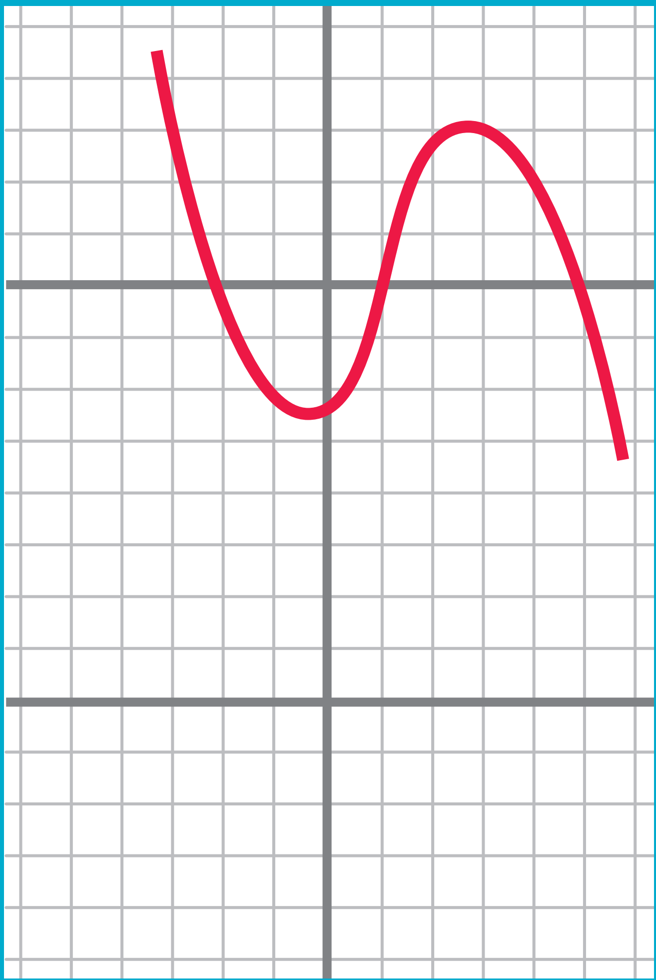
Draw the gradient functions of this curve:



Quick Questions:

Find the equation of the straight-line tangent to the curve $y = 0.2x^2$ at $x = 7$

The graph below shows a gradient function. Sketch its original function below it.



KINEMATICS AND RATES OF CHANGE

Things are beginning to get exciting! Now that we're drawing graphs, we're starting to see how different applications of calculus work! In this section, we're going to look at more ways that NCEA questions will test your differentiation and integration skills – and also how you can use calculus in your everyday life!

We'll be covering:

- Rates of change
- Kinematics
- Kinematics and integration

Rates of Change

Now that we know what rate of change means, let's have a look at how we can find them.

There's two key steps to solving rate of change problems:

- Differentiate the function, since the rate of change is the gradient.
- Substitute the correct value into the gradient function.

Let's straight up dive into an example and see how this works.

The cost of suspending a bridge across the Buller river depends on its length and can be modelled by the formula $C(x) = 400 + 600x^2$, where C is the cost in dollars and x the length in metres.

Calculate the rate at which the cost increases per metre of bridge when the gap that needs to be spanned is 100m wide.

What's our first step? *Differentiate the function*. Cool! What's our function?

$$C(x) = 400 + 600x^2$$

Right. Now let's differentiate: 1200x

Second step? *Substitute in the correct value*. You know exactly which value it's going to be because the question will always say something like 'find *blah blah* when x is *blah blah*'. Your letter might not always be x, but it serves the same purpose.

Ours is *when x is 100m*. So we substitute x = *100* into our gradient function to get the rate of change at the correct moment:

$$C'(x) = \$120000/m$$

Done! The cost increases at $120000 per metre of bridge. Crazy. I wonder what's so important on the other side that they need to reach?

STOP AND CHECK:

The time to set up and record a musician singing a song is given by the equation

$$N = 9t^2 + 3$$

Where $t \geq 0$ and t is the time in seconds and N is the number of words being sung. At what rate are the words being sung after 30 seconds?

Kinematics

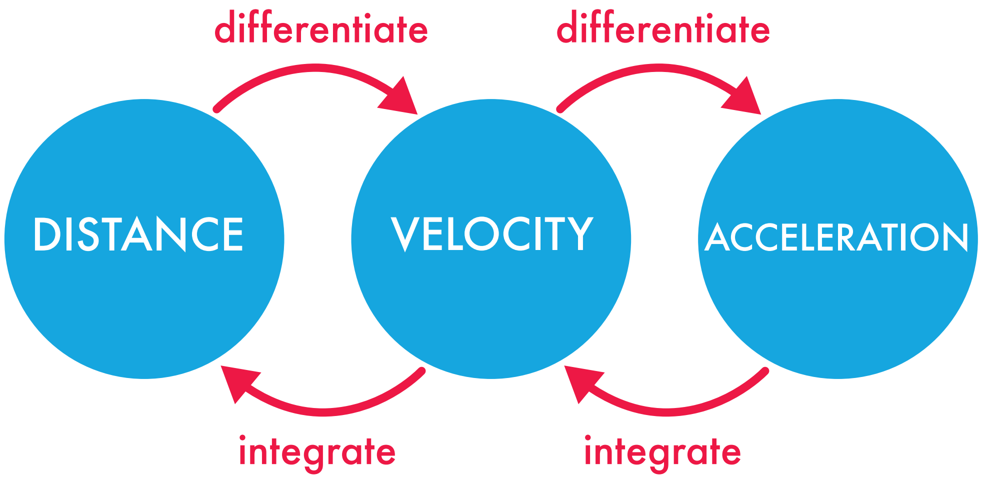
Kinematics is another incredibly useful application of calculus!

If you've studied physics, you'll know that kinematics are all about the link between *distance, velocity,* and *time*. There's two special rates of change that link movement and time.

- *Velocity* is the rate of change of *distance* with respect to *time*, so we can write $v = \frac{ds}{dt}$. This is the first derivative, or the derivative of distance.
- *Acceleration* is the rate of change of *velocity* with respect to *time*, so we can write $a = \frac{dv}{dt}$. This is the second derivative, or the derivative of velocity.

In fact, we can convert between distance, velocity and acceleration simply by using the concepts of differentiation and integration.

One of the easiest ways to conceptualise this is using the diagram:



Kinematics involves a lot of fancy language with its own special symbols, so here's a little cheat sheet for you:

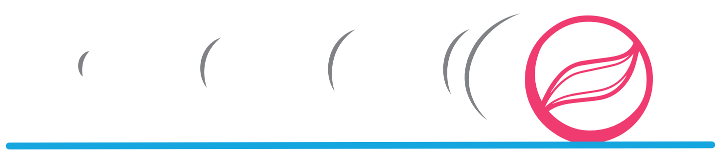
Type	Unit
s = distance v = velocity a = acceleration t = time	*Measured in metres (m)* *Measured in metres per second (m/s)* *Measure in metres per second per second* (m/s^2) *Measured in seconds (s)*

As well as this, here are some handy tricks to use when questions give you numbers instead of explaining how an object is moving:

> $s = 0$ means the object is at the origin (no distance moved).
> $v = 0$ means the object is momentarily at rest (stationary).
> $a = 0$ means the object is travelling at a constant velocity.

Kinematic example

A marble rolls along a straight line so that after t seconds, its distance in metres from its starting point is given as $s(t) = t^2 + 5t$*. What is the velocity of the marble at the end of 4 seconds?*



Alright. So we *know* the distance from the starting point, which is $s = t^2 + 5t$. We *need* the velocity of the marble when $t = 4$ *seconds*.

Looking at the kinematic diagram, we know to *use* differentiation on the equation for distance to get the equation for *velocity*.

$$s = t^2 + 5t$$

Now, differentiate!

$$v = 2t + 5$$

Time to substitute in your time value, which for us is $t = 4$ seconds.

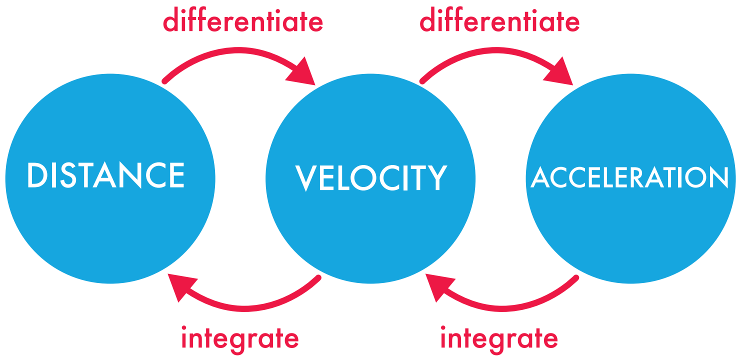
$$v = 2(4) + 5$$

$$v = 13m/s$$

Remember time is usually measured in *seconds*, so remember to convert minutes to seconds if you're given information in minutes.

Integration and Kinematics:

Let's have a look at that chart again:



You're used to going forwards along the chart, but not so much *backwards*. It's a very similar process. You'll use integrals here as well – like in this problem:

The velocity of an object in metres per second after t seconds can be given by the equation

$$v = t^2 + 3t - 1$$

Where $0 \leq t \leq 5$. How far from the starting point is the object after 4 seconds?

Integrate to:

$$s = \int t^2 + 3t - 1 \, dx$$

$$s = \frac{t^3}{3} - \frac{3t^2}{2} - t + c$$

$$s = 41\text{m or } 0.33\text{m}.$$

it has travelled 41m or 0.33m backwards from its starting point

STOP AND CHECK:

- The velocity of a cork floating on a wave is given by the equation $v = 2t - 2$, where v is the velocity in ms^{-1} and t is the time in seconds. If it starts from stationary, how far from the starting point has the cork travelled after 10 seconds?'

Quick Questions:

- A long trench is being dug outside the War Memorial to commemorate the 100th anniversary of ANZAC day. The volume, V, in m^3 of soil removed in t hours by a team of workers can be represented by the equation

$$V = 12t - \frac{2t^2}{5}$$

Where $0 \leq t \leq 15$.

At what rate in m^3/h is the soil being removed at the end of 8 hours?

- The acceleration of a model car is given by $a = 20 - 2t$ for $0 \leq t \leq 10$, where acceleration is in ms^{-2} and t is the time in seconds. Find the velocity at 10 seconds given that $v = 0$ at $t = 0$.

KEY TERMS

Differentiation:
The process of finding a gradient function.

Function:
An equation or a formula. When we say f(x), it means the function of x, so basically, 'an equation about x'.

Gradient:
The slope or steepness of a graph, the rate that x is changing compared to y.

In terms of:
An equation 'y' in terms of 'x' could look like: $y = 2x + 12$ (notice how 'y' is isolated and 'x' is on the other side of the equals sign). One variable is expressed as ONLY a function of the one it is 'in terms of'.

Indices:
A symbol telling you to multiply a value by itself a set number of times.

Integration:
The process of finding a function using a gradient function at a specific point.

Intercept:
The point where a line crosses an axis.

Origin:
The location where the object hasn't moved any amount of distance.

Rate:
The measure of 'how much' something changes or stays constant.

Stationary:
Not moving or 'at rest'.

Substitution:
The process of putting a value or part of equation into another equation.

Tangent:
A line that touches a curve at one point.

Turning Point:
The spot between a positive and a negative gradient, which is flat and so has a gradient of zero

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